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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)  Experiments were conducted with the objective of studying the dynamic behavior of an ice-breaker hull moving continuously through an ice sheet. Of particular interest was the interaction of ice forces exerted against the hull and the hull's pitching motion. A 1:48-scale model hull of the USCGC "Polar Star" (WAGB10) was used in the study. The model hull was towed by a towing carriage.  cont.		

The dynamic response of the model hull to ice breaking was used to infer the vertical component of the ice forces exerted against the hull; to evaluate the temporal variation of the hull's buoyancy and pitching moment; and, to evaluate the inertia force components of the hull's motion. Examination of the dynamic response of the model hull required computer-based acquisition of temporal records of its pitch and draft as well as its vertical and angular accelerations. Time series analyses of the temporal records were performed.

It was found that, as a simplified but reasonable approximation, the dynamic response of an ice-breaker hull moving at constant velocity can be considered as a forced oscillation. It is characterized by the ratio of the frequency  $V_0/l_c$  to the hull's natural frequencies of pitching and heaving, which for the scale model were found to be about the same value,  $f_0$ . The frequency  $V_0/l_c$  is associated with the cycle of ice forces that is experienced by a hull moving with a velocity  $V_0$  through a sheet of ice with a characteristic length  $l_c$ . The mean spacing between the consecutive cracks developed in the ice sheet by the hull is proportional to  $l_c$ ; consequently the parameter  $V_0/l_c$  is a measure of the dominant frequency of ice forces.

For relatively low velocities,  $V_0/f_0 l_c \ll 1$ , the force terms related to a hull's buoyancy are largely in equilibrium with ice forces against a hull, and the inertia forces are negligible. If  $V_0/f_0 l_c = 1$ , the hull is in a state of resonance. As the buoyancy-force components are 180-degrees out of phase with the inertia force components and the two sets of forces partly compensate for each other. At relatively high velocities of hull motion,  $V_0/f_0 l_c \gg 1$ , a superposition of two conditions occurs; the inertia forces compensate the high-frequency components of the ice forces against the hull, and buoyancy forces are negligible; and transient motions of the hull are stimulated at the frequency  $f_0$ .

The bow of a hull moving at low velocities through an ice sheet depresses the ice sheet, which then becomes locally flooded. However, for a hull travelling at relatively high velocity, there may not be sufficient time for the ice sheet to become flooded. Consequently, the hull experienced reduction in buoyancy in the region of its bow, and an increase in the ice forces. The reduction in buoyancy and increase of vertical ice-forces were estimated from the temporal mean values of forces and moments acting on the hull. For the model hull, it was estimated that the depression of the water level was of the order of 0.01 m, which accounted for about one half of the total reduction in the hull's displacement. The reduction in buoyancy force is associated with both increased vertical ice forces against the hull and increased ice-hull friction.

# DYNAMIC RESPONSE OF AN ICE-BREAKER HULL TO ICE BREAKING

by

A. Mueller and R. Ettema

Sponsored by  
Office of Naval Research  
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Iowa Institute of Hydraulic Research, *university of Iowa,*  
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## LIST OF SYMBOLS

$A$	amplitude
$A_w$	area of the hull's waterplane
$d\tilde{A}$	normal to surface element
$A_1$	representative area of depression of water level at the hull's bow
$b$	beam of hull
$b_c$	width of cantilever beam
$\tilde{B}, \Delta\tilde{B}, \Delta B_x, \Delta B_z$	buoyancy force, and increments thereof
$\Delta B'$	reduction of buoyancy force due to depression of water level at bow
$c_1, c_2$	damping coefficients for heave and pitch
$C_{IB}$	coefficient of resistance due to breaking of ice
$C_{IS}$	coefficient of resistance due to submergence of ice
$C_v$	coefficient of resistance due to velocity
$C_w$	coefficient of open-water resistance
$C_o$	co-spectrum
$E_f$	elastic modulus for flexure of ice sheet
$E$	voltage output of velocity meter
$\Delta E$	change in voltage
$f$	frequency
$Fr = \frac{V_o}{\sqrt{gL_s}}$	Froude number of the ship
$F, \bar{F}, \bar{F}_m, F_x, F_z$	ice forces
$F_{zo}$	amplitude of forcing function
$f_h, f_p$	natural frequencies of heave and pitch
$f_o \approx f_h \approx f_p$	natural frequency for heaving and pitching of the model hull

$f_d$	sampling frequency
$\Delta f = 1/T_s$	resolution band width of spectra
$g$	acceleration due to gravity
$GM$	Metacentric height
$G_x(n\Delta f)$	one-sided power spectrum
$G_{xy}(n\Delta f)$	one-sided cross spectrum
$H$	draft of hull
$h$	ice thickness
$H_R(x)$	height of rails of carriage
$I$	mass moment of inertia
$I_v$	virtual mass moment of inertia
$I_{yy}$	area moment of inertia of the waterline for pitching
$k$	radius of gyration
$L, L_I$	moment arm of forces
$L_s$	length of ship
$l_c$	characteristic length of ice sheet
$l_o$	length of pendulum
$l_1, \Delta l$	moment arms to determine length of pendulum
$m$	mass of ship
$m_v$	virtual mass for vertical motion
$m_a$	mass in accelerometer
$\Delta m$	mass increment to calibrate accelerometer
$M_y$	y component of moment of ice forces about hull's center of gravity
$\bar{M}_{ym}$	mean measured value of $M_y$
$\Delta M_{yB}$	moment due to buoyancy forces

$\Delta M'_Y$	moment due to depression of water level at bow
N	number of segments
n	index
P	fracture load of cantilever beam
$P_H$	hydrostatic pressure
$P_I$	ice pressure
Quad	Quad-spectrum
$\tilde{R}$	resultant force on ice-breaker hull
$R_I$	resistance due to breaking of ice
S	wetted area of hull
s	index
t	time
$\tilde{T}, T_x$	thrust of propeller
$T_e$	equilibrium temperature of urea ice
$T_S = 3.84$ sec	length of time segments for calculation of spectra
T	record length of time series
$V_O$	hull velocity
$\tilde{V}$	velocity vector of ice on ship hull
$V(t)$	carriage velocity
$\tilde{W}$	weight of ship
$x_S(t)$	time series
$X_S(t)$	Fourier transform of $x_S(t)$
$x(t)$	position of hull
x	coordinate axis from stern to bow of the hull
y	transverse coordinate



$z$	vertical coordinate
$\Delta z$	average depth of the depression of water level at bow
$z_0$	peak-to-peak value of fluctuations of draft
$\ddot{z}_0$	peak-to-peak value of fluctuations of vertical acceleration
$\alpha$	phase shift
$\alpha_1 \alpha_2 \alpha_3$	coefficients
$\gamma^2$	coherence function
$\delta$	displacement of ice sheet
$\theta$	pitch angle (positive bow up)
$\theta_0$	peak-to-peak value of fluctuation of pitch
$\ddot{\theta}_0$	peak-to-peak value of fluctuations of angular acceleration
$\nu$	Poisson's ratio for ice
$\nu_k$	kinematic viscosity of water
$\rho_w$	density of water
$\rho_I$	density of ice
$\gamma_{xy}$	correlation coefficient
$\sigma_f$	flexural strength of ice sheet
$\tilde{\tau}_v dA$	differential viscous friction
$\tilde{\tau}_I dA$	differential ice friction
$\phi$	roll angle
$\omega$	radial frequency
—	time mean
$\sim$	vector
$\cdot$	derivative with respect to time
$m$	as a subscript, measured value

## I. INTRODUCTION

The use of physical modelling to obtain reliable predictions for the mean resistance forces that are encountered by an ice-breaker hull moving through ice is an open field for research. An improved understanding of the complex dependence of the ice-related resistance force  $R_I$  on the parameters of the ice sheet, on the dimensions of the hull and on its motion requires better knowledge of ice-breaking mechanics. This report presents the results of an experimental study in which more parameters were measured than is customary in testing practice. One of the study's objectives was to explore whether new insights could be derived from such additional information. In particular, vertical forces due to ice breaking, buoyancy and inertia as well as the moments involved in pitching motion of the hull were all included in the analysis of its dynamic response to ice breaking.

Scaling laws provide the relations with which to scale up results of model tests to prototype values. Dimensional analysis is the standard approach used to relate the resistance force  $R_I$  to key parameters involved in the motion of an ice-breaker hull through ice. The parameters are marshalled as groups of dimensionless variables which can then be associated with contributing ice-resistance forces. This approach was taken by Kashteljan (1968), Lewis and Edwards (1970), Edwards et al. (1972), Enkvist (1972), Vance (1975) and others. Edwards et al. (1972), for example, equated the resistance force  $R_I$  to three, ice-related, resistance forces, which they determined from the results of model and full-scale tests of the USCGC "Mackinaw";

$$R_I = C_{IB} \sigma_f b h + C_V \rho_w b h V_O \sqrt{gh} + C_{IS} \rho_w g b h^2 \quad (1)$$

resistance due to flexural failing of ice sheet      resistance due to passage of the hull through broken ice (velocity-dependent resistance)      resistance due to interaction with broken ice floes (gravity resistance)

By dividing  $R_I$  by  $\rho_w g b h^2$ , (1) can be rewritten in the following nondimensional form:

$$\frac{R_I}{\rho_w g b h^2} = C_{IB} \frac{\sigma_f}{\rho_w g h} + C_V \frac{V_O}{\sqrt{gh}} + C_{IS} \quad (2)$$

where  $\rho_w$  = density of water;  $\sigma_f$  = flexural strength of ice;  $b$  = beam of the hull;  $g$  = acceleration due to gravity;  $h$  = ice sheet thickness;  $V_O$  = velocity of ship; and,  $C_{IB}$ ,  $C_{IS}$  and  $C_V$  are coefficients.

A regression analysis can be used to provide the values of the three coefficients  $C_{IB}$ ,  $C_{IS}$  and  $C_V$  and to obtain a set of prototype-scale data from model-scale tests. The three coefficients are assumed to be independent of the dimensionless variables in (2). For a more detailed discussion of the application of dimensional analysis to the study of the resistance to motion of an ice-breaker hull moving continuously through an ice sheet, the reader is referred to the aforementioned studies and also to the studies by Schwarz (1974), Poznyak (1981) and Enkvist (1972, 1983). The latter studies also contain descriptions of experiments that were conducted to determine the contributions to  $R_I$  that are made by the ice-related resistance components.

Resistance to hull motion through level ice sheets can be examined more directly by analyzing the physics involved in ice breaking. White (1969) analyzed vertical and horizontal forces and the moments involved in the ice-hull interaction. Enkvist (1972) gave individual estimates of the contributions of the different components to the total horizontal resistance force,  $R_I$ . Milano (1973, 1975) tried to model all phases of the ice-force cycle for a hull moving through a level ice sheet and proposed a numerical model for predicting  $R_I$ . The present study follows essentially the same lines of analysis that were initiated by these authors.

An ice-breaker hull moving through a level ice sheet breaks ice by applying a vertical force to the ice sheet. This force builds up to the fracture point associated with the flexural strength of ice, subsequently relaxes, then increases until the ice sheet fails once again. This cyclic, or time-dependent, force interacts with the pitch oscillation of the hull. In order to properly understand the dynamic behavior of an ice-breaker hull, it is necessary to consider vertical forces and the pitch moments of the hull as well as horizontal forces. In the present study, a scale model of an ice-breaker hull was instrumented so that its vertical, horizontal and pitching motions could be monitored. This was accomplished by recording the horizontal force imparted to the hull; the draft and pitch angle of the hull; and the vertical and angular accelerations of the hull. From these data, the behavior of the vertical forces imparted to ice sheets by a model ice-breaker hull were estimated as a function of hull velocity. Experiments were conducted using a 1:48-scale model hull of the United States Coast Guard ice-breaker ship USCGC "Polar Star" (WAGB10).

## II. ANALYSIS OF FORCES AND MOMENTS ON AN ICE-BREAKER HULL

The following simplified analysis of forces and moments on an ice-breaker hull outlines the ideas of experimental design and data interpretation that were adopted for the study. The analysis is applied to a hull moving at constant velocity through a level ice sheet.

Here it is noted that differences exist between the dynamic response of hulls which are towed at constant velocity and self-propelled hulls. For the latter, velocity may not be constant during a cycle of ice forces. Also, the pitching amplitude experienced by a self-propelled hull may be less than for a hull moving with constant velocity through a level ice sheet.

Further, because the ice-breaking pattern of a hull may be influenced by hull shape, so the ice-force cycle experienced by a hull may be affected by hull shape. Rolling oscillation of a hull may also affect the ice-breaking pattern, or cycle of ice forces, especially when the natural frequency of roll is close to the frequency of the ice-force cycle.

An additional consideration is the influence of the material behavior of ice on the cycle of ice forces. As was pointed out by Milano (1982), the cycle of ice forces may vary for changing hull velocity because the deformation behavior of ice may change.

### A. General Considerations

The resistance to motion of an ice-breaker hull is the horizontal component of the total force  $\tilde{R}$  due to hydromechanical, viscous and ice forces acting on the hull. The total force is the integral of the normal and tangential stresses acting on the surface areas of the hull, and can be written in vector notation as

$$\tilde{R} = \iint_S (p_H + p_I) d\tilde{A} + \iint_S (\tilde{\tau}_V + \tilde{\tau}_I) dA \quad (2.1)$$

where  $p_H$  = hydrostatic and hydrodynamic pressure;  $p_I$  = ice pressure; and,  $d\tilde{A}$  = elemental area. The term  $\tilde{\tau}_V dA$  is the viscous stress which is negligibly small compared to the shear stress  $\tilde{\tau}_I dA$  attributable to friction between ice and the hull. The shear stress  $\tilde{\tau}_I$  can be expressed as

$$\tilde{\tau}_I = F_{ID} \tilde{V} p_I \frac{\tilde{V}}{|\tilde{V}|} \quad (2.2)$$

where  $\tilde{V}/|\tilde{V}|$  is the unit vector in the direction of the ice movement on the surface element of the hull and  $F_{ID}$  is the coefficient of dynamic friction which may be dependent on the relative velocity,  $\tilde{V}$ , of the hull and ice.

The total resistance force  $\tilde{R}$  is in equilibrium with the thrust  $\tilde{T}$  of the propeller as well as with the weight  $\tilde{W}$  and the inertia forces of the hull (Figure 1a). The equation of motion of the hull can be written, in vector notation, as

$$m_v \ddot{\tilde{X}}_{cg} = \tilde{R} + \tilde{W} + \tilde{T} \quad (2.3)$$

where  $m_v$  = virtual mass of the moving hull; and  $\ddot{\tilde{X}}_{cg}$  = acceleration of the hull's center of gravity. The corresponding moments of the forces  $\tilde{R}$ ,  $\tilde{W}$ , and  $\tilde{T}$  have also to be in equilibrium with the net moment of the inertia force acting on the hull.



If the hull is floating in open water at a vertical position  $z = 0$  (see Figure 1a), the hydrostatic forces  $\tilde{B}(z=0)$  are in equilibrium with the weight  $\tilde{W}$  of the hull,

$$\tilde{W} + \tilde{B}(z=0) = 0 \quad (2.4)$$

If the bow of the hull rides up onto the ice sheet (Figure 1b), it places part of the hull's weight on the ice sheet and the hydrostatic force is changed by  $\Delta\tilde{B}$ ;

$$\tilde{B}(z) = \tilde{B}(z=0) + \Delta\tilde{B} \quad (2.5)$$

The ice force  $\tilde{F}_I$  can then be separated from the total resistance force  $\tilde{R}$  and written as

$$\begin{aligned} \tilde{F}_I &= \tilde{R} - \tilde{B} = \iint_S p_I d\tilde{A} + \iint_S F_{ID} p_I \frac{\tilde{V}}{|\tilde{V}|} dA \\ &= \tilde{R} + \tilde{W} - \Delta\tilde{B} \end{aligned} \quad (2.6)$$

so that the vector sum of total force  $\tilde{R}$  and weight  $\tilde{W}$  becomes

$$\tilde{R} + \tilde{W} = \tilde{F}_I + \Delta\tilde{B} \quad (2.7)$$

Note that (2.6) and (2.7) neglect all hydrodynamic forces associated with viscous drag and wave resistance to hull motion.

The complex spatial and temporal distributions of  $p_I$ ,  $F_{ID}$  and  $\tilde{V}$  are not known and cannot be measured by way of a simple experiment. As an alternative approach, considering only the ice forces and hull motion in the two-dimensional  $x$ - $z$  plane (see Figure 1b), the ice forces,  $F_x$ ,  $F_z$  and their corresponding moment about the  $y$ -axis,  $M_y$ , can be estimated from the dynamic response of the hull if the buoyancy force increment  $\Delta\tilde{B}$  and its corresponding moment increment  $\Delta M_{yB}$ , associated with pitching and heaving of the hull, can be estimated from the hull's position in the  $x$ - $z$  plane.

### **B. Estimation of Buoyancy Forces**

The equation of motion, (2.3), of a hull moving with constant horizontal velocity along the  $x$ -axis can be written as

$$m_V \ddot{x}_{cg} = 0 = F_x + \Delta B_x + T_x \quad (2.8)$$

and

$$m_V \ddot{z}_{cg} = F_z + \Delta B_z \quad (2.9)$$

The equation for the pitching motion of the hull is

$$I_V \ddot{\theta} = M_y + \Delta M_{yB} \quad (2.10)$$

where  $I_V$  = virtual mass moment of inertia; and  $\ddot{\theta}$  = angular acceleration. The temporal mean values of  $\ddot{z}_{cg}$  and  $\ddot{\theta}$  are zero, since heave and pitch motions are limited. The acceleration  $\ddot{x}_{cg}$  is also zero because the model hull is towed at constant velocity.

The hydrostatic pressure distribution acting against the hull is not disturbed at low velocities of hull motion provided the depressed portion of the ice sheet at the hull's bow is flooded with water. At higher hull velocities this may no longer be true because there may not be sufficient time for water to flood the depressed portion of the ice sheet. In addition, it has been observed in field and model tests (e.g., Enkvist 1972) that a ventilated area may form between a high velocity bow and the depressed portion of an ice sheet. The disturbance to the distribution of hydrostatic pressure and the depression of the water surface level at the hull's bow may lead to a reduction of the buoyancy force  $\Delta B_z$  and give rise to a positive value for the buoyancy force component  $\Delta B_x$ . Consequently, only for relatively low hull velocities (see Figure 1) can the buoyancy force increments  $\Delta B_z$  and  $\Delta B_x$ , and their resulting moment increment  $\Delta M_{yB}$ , be estimated\* using the following relationships:

$$\Delta B_z = - A_w \rho_w g \bar{z} \quad (2.11)$$

$$\Delta B_x = 0 \quad (2.12)$$

and

$$\Delta M_{yB} = - I_{yy} \rho_w g \theta \quad (2.13)$$

---

\*The proportionality constant  $I_{yy} \rho_w$  in (2.13) can be obtained experimentally by measuring trim angle of the model hull due to a known weight at its bow or stern. Similarly, one can determine the proportionality constant  $A_w \rho_w g$  in (2.11).

where  $A_w$  = area of the waterplane of the hull;  $\bar{z}$  = average position of the hull above the equilibrium level  $z = 0$ ;  $I_{yy}$  = area moment of inertia of the waterline for pitching; and  $\theta$  = pitch angle. The additional terms for higher hull velocities can only be crudely estimated since no measurements are available. The additional change in the buoyancy force  $\Delta B'_z$  must be of the order of the reduction of the weight of the displaced fluid; that is

$$\Delta B'_z \approx -A_1 \rho_w g \bar{\Delta z} \quad (2.14)$$

where  $A_1$  and  $\bar{\Delta z}$  are the characteristic area and average depth, respectively, of the depressed portion of the ice sheet. The moment increment  $\Delta M'_y$  can be estimated (Figure 1c) as

$$\Delta M'_y = \Delta B'_z L \quad (2.15)$$

where  $L$  = the moment arm of  $\Delta B'_z$ . The buoyancy force increment  $\Delta B'_x$  is of the order of the vertically-integrated hydrostatic pressure; that is

$$\Delta B'_x \approx b \rho_w g \frac{\bar{\Delta z}^2}{2} \quad (2.16)$$

where  $b$  = width of the beam at the relevant station of the hull. The ratio  $\Delta B'_z / \Delta B'_x$  is large compared with unity because the area of the depressed portion of the ice sheet,  $A_1$ , is large compared with the area  $b \bar{\Delta z}$ ;

$$\frac{\Delta B'_z}{\Delta B'_x} = \frac{A_1 \bar{\Delta z}}{b \bar{\Delta z}^2 / 2} = \frac{2A_1}{b \bar{\Delta z}} \gg 1 \quad (2.17)$$

It follows from the preceeding analysis, (2.11) through (2.17), that the temporal mean values of  $F_x$ ,  $F_z$  and  $M_y$  can be estimated using the following equations:

$$\bar{F}_x = \bar{T}_x + \Delta B'_x \quad (2.18)$$

$$\bar{F}_z = A_w \rho_w g \bar{z} + A_l \rho_w g \bar{\Delta z} \quad (2.19)$$

and

$$\bar{M}_y = I_{yy} \rho_w g \theta + \Delta B'_z L \quad (2.20)$$

The force  $\bar{F}_x$  is approximately equal to the towing force  $\bar{T}_x$  because the contribution of the buoyancy term,  $\Delta B'_x$ , is small. The force  $\bar{F}_z$  is increased by the depressed water level at the bow. The vertical force due to the mean change in the hull's draft,  $\bar{z}$ , by itself is an underestimate of  $\bar{F}_z$ . Part of the buoyancy which had compensated for the hull's weight is replaced by an ice force. Since an increase in ice forces against the hull causes a larger friction force to be exerted against the hull,  $\bar{F}_x$  and  $\bar{T}_x$  are, therefore, also increased by the depression of the water level. The moment  $\bar{M}_y$  is underestimated when it is equated solely to the term containing the pitch angle  $\theta$ , if the water level of the bow is depressed at higher velocities. For the purpose of the interpretation of the experimental results, it is assumed that the distribution of hydrostatic pressure around the hull is undisturbed by the deflection of the ice sheet. The validity of this assumption is to be

checked from the results of experimentation and estimates of  $\Delta B'_z$ ,  $\Delta B'_x$  and  $\Delta M'_y$  are attempted by making use of relations discussed in the following section.

### C. Estimation of Ice Forces

The components  $F_x$  and  $F_z$  of the resultant ice force  $F_I$  are components of the same distribution of the ice pressure against the hull. Because integration of (2.6) for each force component involves the same surface area of the hull, the two force components have the same scale and their ratio can be assumed to be approximately constant for each hull velocity,

$$\frac{F_z}{F_x} = \alpha_1 \quad (2.21)$$

This assumption is valid so long as both the distribution of ice pressure and the directions of ice-rubble movement around the hull are similar from test to test.

The line of action of the ice force usually acts through the hull's bow. If the contribution of the horizontal ice force,  $F_x$ , to the pitching moment,  $M_y$ , is small compared with the contribution of the vertical ice force  $F_z$ , the length of the moment arm  $L_I$  is equal to the distance of the line of action of  $F_z$ , where it crosses the waterline, to the hull's center of gravity;

$$L_I = \frac{M_y}{F_z} \quad (2.22)$$

The moment arm  $L_I$  has to stay within one half of the ship length.



The vertical ice force,  $F_z$ , is produced by the buoyancy of the depressed and submerged ice and the bending force needed to break the ice sheet;

$$F_z = \alpha_2 A_w (\rho_w - \rho_I) g h + \alpha_3 \sigma h^2 \quad (2.23)$$

where  $\alpha_2$  and  $\alpha_3$  are constants of the order of unity. The first term in (2.23) can be assumed to be constant with time, while the second term is time-dependent as it represents the increase and relaxation of the bending forces in the ice sheet. Inertial forces of the ice floes and the water moving around the hull, which may be important at high velocities of hull motion, are not included in this estimate.

#### D. Interaction of Ice Forces with Heaving and Pitching Motions of a Hull

The equation of heaving motion for a hull can be written as

$$\begin{array}{ccccccc} m_v \ddot{z} & + & c_1 \dot{z} & + & \rho_w g A_w z & = & F_z(t) \\ (i) & (ii) & (iii) & & (iv) & & \end{array} \quad (2.24)$$

where  $m_v$  = virtual mass for vertical motion;  $A_w$  = area of the waterplane of the ship; and,  $c_1$  = damping coefficient. The linearized equation for pitching of a hull can be written as

$$\begin{array}{ccccccc} I_v \ddot{\theta} & + & c_2 \dot{\theta} & + & \rho_w g I_{yy} \theta & = & M_y(t) \\ (i) & (ii) & (iii) & & (iv) & & \end{array} \quad (2.25)$$

where  $\theta$  = pitch angle;  $I_v$  = virtual mass moment of inertia;  $I_{yy}$  = area moment of inertia of the waterplane about the y-axis;  $c_2$  = damping coefficient;

and,  $M_y(t)$  = moment of the ice forces relative to center of gravity of the hull.

Both heaving and pitching motions of a hull can be analyzed as forced oscillations. The inertia terms, (i), the damping terms, (ii), and the buoyancy terms, (iii), are in equilibrium with forcing functions, (iv). The natural frequencies of heaving and pitching of a hull are  $f_h$  and  $f_p$ , respectively, where

$$f_h = \frac{1}{2\pi} \left( \frac{\rho_w g A_w}{m_v} \right)^{0.5} \quad (2.26)$$

and

$$f_p = \frac{1}{2\pi} \left( \frac{\rho_w g I_{yy}}{I_v} \right)^{0.5} \quad (2.27)$$

For the model hull of the USCGC Polar Star, it was determined that  $f_h$  and  $f_p$  are approximately 1Hz and are both hereinafter referred to as  $f_o$ . The parameters  $m_v$ ,  $I_v$ ,  $A_w$ , and  $I_{yy}$  are properties of the hull and were estimated in accordance with their open-water values. The damping coefficients,  $c_1$  and  $c_2$ , for ice are as yet unknown. However, it is of interest to examine the form of a solution of (2.24) and (2.25), both of which include damping terms.

With a forcing function in the form of  $F_z(t) = F_{zo} \exp(i2\pi ft)$ , (2.24) has the following steady-state solution:

$$z(t) = \frac{1}{4\pi^2} \frac{F_{zo}}{(m_v^2 (f_o^2 - f^2)^2 + c_1^2 f^2)^{0.5}} \exp(i(2\pi ft - \alpha)) \quad (2.28)$$

where

$$\alpha = \arctan \frac{2\pi f_c}{4\pi^2 m_V (f_0^2 - f^2)} \quad (2.29)$$

The second derivation of  $z(t)$  with respect to time is

$$\begin{aligned} \ddot{z}(t) &= -4\pi^2 f^2 z(t) \\ &= -\frac{F_{ZO} f^2}{(m_V^2 (f_0^2 - f^2)^2 + c_1^2 f^2)^{0.5}} \exp(i(2\pi ft - \alpha)) \end{aligned} \quad (2.30)$$

The ratio of the inertia term  $m_V \ddot{z}(t)$  and the buoyancy term  $\rho_w g A_w z(t)$  is

$$\frac{m_V \ddot{z}(t)}{\rho_w g A_w z(t)} = -4\pi^2 f^2 \frac{m_V}{\rho_w g A_w} = -\frac{f^2}{f_0^2} \quad (2.31)$$

The inertia and the buoyancy terms are 180 degrees out of phase for all frequencies of pitching and heaving of the hull. At resonance,  $f$  equals  $f_0$ , so that both terms have the same amplitude. At low frequencies, when  $f$  tends to zero, the buoyancy term  $\rho_w g A_w z$  is in equilibrium with  $F(t)$  and the inertia term is negligible. At high frequencies, when  $f$  tends to infinity,  $m_V \ddot{z}$  tends to  $F(t)$  and the buoyancy term is negligible. The phase angle,  $\alpha$ , of  $z(t)$  relative to the forcing function,  $F(t)$ , is 90 degrees at resonance. It tends to zero at low frequencies and to 180 degrees at high frequencies, for which  $\ddot{z}(t)$  is in phase with the forcing function.

In addition to the above steady-state solution, the transient solution which is the response to an impact in  $z$ , or  $\ddot{z}$ , has the form

$$z(t) = A \frac{c_1}{m_V} \exp(i2\pi (f_0^2 - \frac{c_1^2}{16\pi^2 m_V^2})^{0.5} t) \quad (2.32)$$

where the amplitude,  $A$ , depends on the initial conditions of  $z$  or  $\ddot{z}$ . Physically, (2.32) relates to a damped oscillation at a fixed frequency which is determined by the resonant frequency  $f_0$  together with the ratio of the damping coefficient and the mass  $m_V$ . The ratio of the inertia and the buoyancy terms is close to unity if damping is small;

$$\frac{m_V \ddot{z}(t)}{\rho_w g A_w z(t)} = \frac{1}{f_0^2} (f_0^2 - \frac{c_1^2}{16\pi^2 m_V^2}) \leq 1 \quad (2.33)$$

Analogous relations can be formulated for  $I_V \ddot{\theta}$  and  $\rho_w g I_{YY} \ddot{\theta}$ .

If the draft  $z(t)$ , the pitch angle  $\theta(t)$ , as well as their derivatives with respect to time,  $\dot{z}(t)$  and  $\dot{\theta}(t)$ , are measured as functions of time, information on the ice forces for the entire of frequency band can be gained by Fourier analysis. A direct evaluation of the sum of the damping term and the ice forces (terms (ii) and (iv) in (2.24) and (2.25)) is, however, difficult because the phases of  $z(t)$  and  $\dot{z}(t)$  with respect to  $\theta(t)$  and  $\dot{\theta}(t)$  must be measured with high accuracy.

The temporal behaviors of  $I_z(t)$  and  $M_y(t)$  are governed by the breaking pattern of the ice sheet. During impact with the ice sheet, the hull rides onto the ice sheet until the sheet fails. When this occurs, the vertical ice force acting through the hull relaxes. The radius of the resulting approximately circular crack, through the ice sheet in the vicinity of the hull's bow, scales with the ice sheet's characteristic length,  $l_c$ , where

$$l_c = \left[ \frac{E_f h^3}{12 \rho g (1 - \nu^2)} \right]^{0.25} \quad (2.34)$$

and  $E_f$  = elastic modulus for flexure of the ice sheet;  $\nu$  = Poisson's ratio for ice. The term  $l_c/V_0$  is the period of travel between consecutive circular cracks. Consequently, the time-dependent part of the forcing functions  $F_z(t)$  and  $M_y(t)$  can be considered as a series of impacts of different intensities at the preferred frequency of  $V_0/l_c$ . The dynamic response of the hull to ice breaking is thus a superposition of a quasi-steady-state solution oscillating at a frequency of  $V_0/l_c$  and transient solutions starting at strong impacts and oscillating at a frequency of  $((f_0^2 - c_1^2)/16\pi^2 m_v^2)^{0.5}$ .

If  $V_0/f_0 l_c$  equals unity, the pitching of the hull is in resonance with the frequency of ice breaking and the inertia term is approximately equal to the buoyancy term with a 180-degree phase shift. If  $V_0/f_0 l_c$  is significantly less than unity--i.e., for low velocities--buoyancy forces are in equilibrium with ice forces and inertia terms can be neglected. If  $V_0/f_0 l_c$  is much greater than unity--i.e. for high velocities--inertia forces become dominant and are chiefly responsible for ice breaking. Draft and pitch of the hull will adjust to the mean forces only, provided that there are no strong impacts which induce a superimposed transient solution at the resonant frequency.

The foregoing analysis of the dynamic response of an ice-breaker hull to ice breaking does not account for a likely feed-back of hull motion on ice forces; i.e., in the foregoing analysis, the z-axis component of the ice force and the pitching moment,  $F_z(t)$  and  $M_y(t)$ , are both assumed to be independent of  $z(t)$  and  $\theta(t)$ , respectively.

### III. EXPERIMENTATION

#### A. Experimental Facilities

##### **1. The IIHR Ice Towing Tank**

The experiments were conducted in the 20m x 5m x 1.3-meter deep model ice towing tank of the IIHR. Figure 2 is an overview of the towing tank, the cold room in which it is housed, and its cooling system. The cooling system is composed of two compressors which provide coolant to the two cooler units situated at each end of the cold room. The compressors are in turn cooled by water pumped from a 200 m<sup>3</sup> sump. If the sump temperature exceeds a certain limit, a cooling tower situated outside of the building is operated to cool the sump water.

Fans inside the four cooler units draw air from the cold room and, after the heat exchange has occurred discharge it into eight ducts which extend the whole length of the cold room. The chilled air is forced through an array of half-inch diameter holes along the base of each duct, thereby producing a flow of chilled air over the towing tank. The four ducts of each side alternate to provide an even distribution of the cold air. Every two hours, one pair of cooler units is defrosted by electrical heating. Depending on the ambient air temperature outside the cold room, the total cooling capacity of the system varies between 15 and 20 kW, and enables an ice sheet to grow at a rate of 1.5 to 2 mm per hour.

The 5-meter wide x 2.4-meter long motorized carriage, depicted in Figure 3, was used to tow the model ice-breaker hull. The carriage runs along rails on the concrete walls of the towing tank. The vertical level of each rail was

adjusted to a tolerance of  $\pm 1.5\text{mm}$  along its length. An angle beam on one side of the basin gives the lateral guidance and carries the rack of the rack-and-pinion drive mechanism. The D.C. motor on the carriage has a maximum torque of 31 Nm and a speed range of 58 to 1750 RPM. A 1:15 gear box increases the torque to 413 Nm and gives a reduced speed range of 3.9 to 117 RPM. The effective radius of the pinion is 0.06m; consequently the carriage has a maximum driving force of 6800 N and a velocity range of 0.024 m/sec to 0.74 m/sec. Higher velocities, up to 2.2 m/sec, can be achieved if a 1:5 gear box is coupled to the D.C. motor.

In order to measure the velocity of the carriage, a wheel carrying a circular array of holes is mounted on the drive shaft of the D.C. motor. The passage of each hole, as the shaft rotates, is sensed by a photo detector which emits a light through the hole. The number of pulses counted during a time interval is proportional to the velocity of the carriage. The length of the time interval is 0.371 seconds so that 1000 pulses correspond to a velocity of 0.333 m/sec. After each interval of 0.371 sec, the number of pulses is latched to a display and to a digital-analog converter which holds the voltage during the following interval until the next measurement is available. The mean velocity of the preceding interval is therefore, displayed and can be sampled.

## 2. The Model Ice-Breaker Hull

A 1:48-scale model of the USCGC "Polar Star" (WAGB10), loaned to the IIHR by the U.S. Coast Guard, was used to conduct the study. The abbreviated lines of the prototype, and model, hull of the Polar Star are depicted in Figure 4 and its principal model dimensions are given in Table 1.



Care was taken to accurately determine the relevant mechanical constants of the hull in order to accurately evaluate forces and moments due to inertia and buoyancy. The model hull was loaded with a mass of 102.6 kg so that its center of gravity was located, to within a tolerance of 0.014 m, at the longitudinal center of buoyancy. The area  $A_w$  of the model hull's water plane (the plan area of the hull at its water line) was determined to be  $0.81 \text{ m}^2$ . This value was controlled by loading the floating hull with a mass increment  $\Delta m$  and measuring the change in draft  $\Delta z$ . From the relation

$$\rho A_w \Delta z = \Delta m \quad (3.1)$$

the result  $A_w = 0.82 \text{ m}^2 \pm 0.08 \text{ m}^2$ , was obtained.

The hull was suspended as a pendulum so that its moment of inertia could be evaluated in accordance with the equation of motion

$$(\pi k_O^2 + I) \ddot{\theta} + Mg l_O \theta = 0 \quad (3.2)$$

which enables  $I$  to be determined from the length of the pendulum,  $l_O$ , the mass of the hull,  $m$ , and the radial frequency of oscillation,  $\omega$ , where

$$\omega^2 = \frac{mg l_O}{\pi k_O^2 + I} \quad (3.3)$$

The length  $l_O$  was measured by loading the suspended hull with a mass  $m_1$  at its bow (see Figure 5 for a definition of the terms in (3.3)). The equilibrium of moments,

$$m \Delta l = m_1 l_1 \quad (3.4)$$

together with the relation

$$\Delta l = \theta l_0 \quad (3.5)$$

permits  $l_0$  to be determined in terms of  $m$ ,  $m_1 l_1$  and the angle of rotation,  $\theta$ . The resulting moment of inertia,  $I$ , was calculated to be  $33.3 \pm 0.5 \text{ kg m}^2$ , corresponding to a radius of gyration  $k = 0.57 \text{ m}$ ; according to the relation

$$I = mk^2 \quad (3.6)$$

No prototype values were available for comparison with the measured values of  $I$  and  $k$ .

The area moment of inertia,  $I_{yy}$ , for pitching of the hull about the  $y$  axis (Figure 1) was determined by loading the floating hull with a mass at its stern and measuring the resulting pitch angle of the hull. A value of  $I_{yy} = 0.25 \pm 0.02 \text{ m}^4$  was calculated. This value corresponds to a metacentric height  $GM = 2.4 \text{ m}$ . The vertical distances of the water-line to the center of gravity and the center of buoyancy were assumed to be small compared to the metacentric height  $GM$ ;

$$GM \approx \frac{\rho I_{yy}}{m} \quad (3.7)$$

The added mass for vertical motions of the hull was estimated using the method formulated by Landweber and Macagno (1957). They describe the hull for each section as a Lewis form which is based on two parameters; the ratio of beam,  $b$ , to the draft,  $H$ ; and the ratio of the sectional area to the area defined by the product  $bH$ . In this manner, an added-mass coefficient,  $a = 1.38$ , was calculated for the model hull. The added-mass moment of inertia,  $I_v$ , (or the virtual moment of inertia) was estimated by measuring the pitch frequency of the hull. An eccentric driven by a small D.C. motor with a variable speed control was mounted at the stern of the ship hull and operated so that it produced an oscillating vertical force with an amplitude of about 1N. From the equation of motion for hull oscillation, (2.25), the resonant frequency was determined to occur when

$$4\pi^2 f_p^2 = \frac{mg}{I_v} = \frac{mg}{I(1+A)} \quad (3.8)$$

The resonant frequency,  $f_p$ , was determined to be 1.09 Hz, which is associated with an added-mass moment of inertia,  $I_v = 51.5 \text{ kg m}^2$ , or an added-mass moment of inertia coefficient,  $A = 0.55$ . The two coefficients,  $a$  and  $A$ , were estimated for hull motion in open water. They are at best a lower limit for the coefficients of the hull when it is breaking an ice sheet.

The open-water resistance force  $F_{X0}$  was measured as a function of hull velocity. The resistance force  $F_{X0}$  and the resistance coefficient  $C$  are listed in Table 2 together with Froude number,  $V_0/\sqrt{gL}$ , and Reynolds number,  $V_0 L/\nu$ , for each test run. The resistance coefficient  $C$  was estimated as

$$C = \frac{F_{xo}}{0.5 \rho_w V_o^2 S} \quad (3.9)$$

where  $S$  is the wetted area of the hull. As was assumed for the analysis of forces on an ice-breaker (Chapter II), it is evident in Table 2 that the open-water resistance to the hull's motion is small compared with the ice-breaking resistance. Furthermore, the Froude numbers of the hull are too small for the wave resistance to be of significance (Lackenby 1965).

The coefficient of friction for evaluating the friction force acting between ice and hull was measured for saline ice during tests conducted by Arctec Inc. (Lecourt and Deslaurian 1976). For the surface finish of the model hull used at IIHR, the Arctec Inc. study gave  $F_{ID} = 0.478$ . No additional measurement of  $F_{ID}$  was made during the present study.

### 3. Instrumentation

The model hull was connected by a 25-millimeter diameter shaft to a dynamometer supported from the carriage, as is shown in Figure 6. The shaft was connected to a vertical plate of the dynamometer by way of two ball bushings which enabled the shaft, and the hull, to move vertically. To ensure an almost moment-free connection of the hull, the shaft was fixed by a ball bearing to the base of the hull. Friction in the ball bushings affected the vertical motion of the model hull. A measurement of the hysteresis of the vertical position of the hull under zero load resulted in an uncertainty  $\Delta z = \pm 0.65$  mm in its vertical displacement. This value corresponded to a vertical resistance force  $\Delta F_z = \rho_w g A \Delta z \approx 5$  N. Once static friction in the ball bushing

was overcome, the vertical resistance is reduced because friction is reduced to a lower value commensurate with dynamic friction.

The dynamometer, which was used to measure the translational inertia force of the hull, or the towing force, consisted of two 0.15-millimeter thick plates which allowed a horizontal motion that was restricted by a 50 lb Statham UL4-50 load cell instrumented with a Statham UC-3 force transducer. The load cell limited the full-scale displacement of the dynamometer to 0.12 mm. The dynamometer and its supports had a resonant frequency of 6 Hz which was detected in the signal produced by the force transducer. Yawing of the hull was restricted by the location of a vertical rod at its stern. The rod was constrained to slide in a 36-millimeter diameter ring which was connected by a frame (which can be seen in Figure 10).

Angular and vertical accelerations of the hull were measured using accelerometers mounted at its bow and stern. Each accelerometer was formed of a 5 lb UL4-5 Statham load cell loaded with a mass of 2.23 kg in order to sensitize it to vertical acceleration. The range of the force transducers correspond to accelerations of about  $1g$ , or  $9.81 \text{ m/sec}^2$ . The resolution of each accelerometer was of the order of  $0.01 \text{ m/sec}^2$ . This precision could not be further improved since measured acceleration amplitudes were small compared to the maximum load range of  $1g$ . Resonant frequencies of the accelerometers were well beyond the bandwidth of the data acquisition system that was used for the experiments. The accelerometers were sensitive to tilting because a roll of angle  $\phi$  would reduce the vertical load against the hull by an amount

$$\Delta(gm) = gm(1 - \cos\phi) \approx gm \frac{\phi^2}{2} \quad (3.10)$$

This reduction of the load is equivalent to a negative (downward) acceleration of

$$\ddot{z} = - \frac{\Delta (gm)}{m} = - \frac{g\phi^2}{2} \quad (3.11)$$

The accelerometer resolution of  $0.01 \text{ m/sec}^2$  is equivalent to a roll angle of  $\phi = 2.6^\circ$ . In some ice-breaking tests, a low frequency deviation of the vertical acceleration was observed and was attributed to rolling of the hull. Both accelerometers sensed the same roll angle. Therefore, the measured angular accelerations of the hull were not affected by rolling of the hull. Pitch angles were too small to be sensed by the accelerometers.

The force transducer of the dynamometer and the two accelerometers were powered by a bridge amplifier, which was located outside of the ice room.

Heave and pitch motions of the hull were measured by recording, with linear displacement potentiometers, the vertical position of the hull at two positions. The potentiometers were excited with 12 volts, which corresponded to a full stroke movement range of 0.15 m. The outputs of the potentiometers were transmitted by means of two voltage followers.

The six voltages from the load cells, accelerometers, and potentiometers, together with the carriage velocity, were scanned using a digital voltmeter. The digitized data were serially transmitted through a telephone link to the IIHR's HP-1000 computer system and were there stored on disk. The bandwidth of the data acquisition link was 100 Hz so that each channel was sampled at a rate of 16.7 Hz. At this sampling rate, the 6 Hz resonance on the force

signal could be resolved and was filtered digitally at 3 Hz. The signals from the two accelerometers were filtered with a 48-dB/octave analog filter with the 3-dB point set at 6 Hz. The signals of both the linear potentiometers and the carriage velocity did not need filtering before sampling.

## **B. Experimental Procedures**

### **1. Ice Sheet Growth**

Ice sheets were grown from a 1.3-percent, by weight, urea solution according to the following procedure: With the cooler system operating at full capacity, the water in the towing tank was cooled down to a temperature of  $-0.2^{\circ}\text{C}$  and the room was simultaneously cooled to about  $-12^{\circ}\text{C}$ . An air-bubble system provided the necessary mixing of the solution to prevent supercooling of its surface layer. Before being wet-seeded, the surface of the urea solution was screened to remove floating ice crystals. Then, the air bubbles and the blowers of the cooling units were shut off and the cold room was fogged with a fine spray of water droplets. The spray was produced with the use of a pressurized air spray gun and a pressurized tank. The water droplets froze in the cold air and settled onto the surface of the water which had by then reached the freezing temperature of  $-0.4^{\circ}\text{C}$  for the urea solution. The wet-seeding process prevented the formation of relatively large ice crystals and enabled a multitude of small crystals to grow simultaneously over the surface of the urea solution.

Each ice sheet was permitted to grow to about 80 percent of its thickness,  $h$ . The room temperature was then raised to about  $2^{\circ}\text{C}$  and the ice sheet allowed to warm and weaken. The bending strength,  $\sigma_f$ , and the flexural



elastic modulus,  $E_f$ , of the ice sheet were monitored until  $\sigma_f$  reached the values prescribed for the ice-breaker test. The load,  $f$ , to flexurally fail a cantilever beam of length  $l$  and width  $d$  was used as to estimate the value of  $\sigma_f$ , where

$$\sigma_f = \frac{6fl}{dh^2} \quad (3.12)$$

Four to six cantilever beams were failed in order to get a representative mean value of  $\sigma_f$ . The flexural elastic modulus,  $E_f$ , was determined by measuring the increment,  $\delta$ , of the vertical deflection of the ice sheet due to small increments of a point load,  $\Delta P$ , at the center-point of the ice sheet. Thereby

$$E_f = \frac{3}{16} \frac{(1-\nu^2)}{\rho_w g h^3} \left( \frac{\Delta P}{\delta} \right)^2 \quad (3.13)$$

## 2. Calibration of the Transducers

For each of the six channels of data (force, accelerations at bow and stern of the hull, two linear potentiometers and the carriage velocity) the zero level and the sensitivity of the transducer were determined before every run. The sensitivities of the transducers proved to be constant for all the runs.

The sensitivity of the dynamometer was determined to be 33.5 N/Volt. It was measured by means of a horizontal load applied at the stern of the hull. The sensitivities of the accelerometers were determined by measuring the change in output voltage,  $\Delta E$ , due to an applied load increment,  $\Delta F$ , of 1.47 N. The corresponding sensitivity of  $-0.42 \text{ m/sec}^2/\text{Volt}$  equal to  $\Delta F/m_a \Delta E$

where  $m_a$  is the suspended mass in the accelerometer. The sensitivity of the linear potentiometers, reckoned to be 13.0 mm/Volt, was measured as the ratio of a change in length to a change in signal output. The sensitivity of the circuit for the carriage velocity was determined by correlating its output voltage with the mean velocity of the carriage (determined by use of a stop watch and a length scale). Then, the analog setting of the carriage speed control was calibrated against the output voltage.

Using the linear displacement potentiometers to measure the position of the model hull relative to the carriage rails, it was found that, as is stated in Section III.A.1, the level of the carriage rails varied to within a tolerance of  $\pm 1.5$  mm along the length. In order to improve the accuracy of the measurements of hull motion, the position of the rails relative to the water surface had to be determined as a function of the position of the carriage along the ice tank. This was accomplished by slowly towing the freely-floating model hull in open water and observing the change in hull position as was indicated by the potentiometers.

In order to calculate the position  $x(t)$  of the carriage during the ice-breaking tests, the velocity signal,  $V(t)$ , was integrated with respect to time. As is mentioned in Section III.A.3, the voltage output,  $E_v(t)$ , lagged behind the velocity signal,  $V(t)$ , because  $E_v(t)$  was equal to the mean velocity of the preceeding interval. In order to improve the estimate of the mean actual velocity of the  $n$ th time interval, the step in the signal was detected and it was assumed that the change in velocity from interval  $n-1$  to  $n$  was the same as the step intervals  $n-2$  to  $n-1$ . An accuracy of  $x(t)$  of  $\pm 50$  mm was required for the tests. Therefore, the elevation of the rails,  $H_R(x)$ , (Figure

7) was smoothed by a moving-average integration technique applied to 50 mm lengths of rail, so as to be consistent with the resolution of hull position. Table 3 is a summary of the bandwidths, and the sensitivities of the various transducers comprising the instrumentation. The errors of the various measured quantities are listed in Table 4.

### 3. Conduct of the Experiments

In order to get the model hull prepared for each test, a rectangular slot was cut in the ice sheet while it was warming and weakening. With the hull in open water, zero levels for the six data channels were recorded and the sensitivities of the transducers were checked. For each ice sheet, data were taken for an initial run of about two hull lengths, with the hull moving at a low velocity of 0.03 m/sec. Subsequently, a second run was executed with the velocity set as is indicated in the list of parameters given in Table 5. In addition to recording the motion of the hull on video tape, photographs of the hull were taken during each test. At the completion of a test, the ice fragments in the channel behind the ship were also photographed.

### 4. Numerical Methods

Mean,  $\bar{x}$ , and standard deviation,  $\sigma_x$ , of a time series  $x(t)$  of record length  $T$  and sample rate  $f_d$  were calculated using the relationships

$$\bar{x} = \frac{1}{f_d T} \sum_{n=1}^{T f_d} x(n/f_d) \quad (3.14)$$

and

$$\sigma_x = \left[ \frac{1}{f_d T} \sum_{n=1}^{T f_d} x^2(n/f_d) - \bar{x}^2 \right]^{0.5} \quad (3.15)$$

The covariance,  $\sigma_{xy}$ , of two time series  $x(t)$  and  $y(t)$  was determined using

$$\sigma_{xy}^2 = \frac{1}{f_d T} \sum_{n=1}^{T f_d} x(n/f_d) y(n/f_d) - \overline{xy} \quad (3.16)$$

The correlation coefficient for two time series  $x(t)$  and  $y(t)$  is

$$\rho_{xy} = \frac{\sigma_{xy}^2}{\sigma_x \sigma_y} \quad (3.17)$$

Power spectra, cross spectra and coherence functions were calculated by way of the following methods (e.g., see Bendat and Piersol 1971):

The time series were split into  $N$  segments each with  $M = 64$  samples or segment period  $T_s = M/f_d = 3.84$  seconds. For each segment, a Fourier series was calculated with a resolution bandwidth  $\Delta f = 1/T_s = 0.26$  sec. Frequency leakage was reduced by subtracting the time mean and by a cosine taper window.

Each tapered segment  $x_s(t)$  of the signal was then represented by

$$x_s(t) = \sum_{n=-M/2}^{M/2} \frac{1}{T_s} X_s(n\Delta f) \exp(i2\pi n\Delta f t) \quad (3.18)$$

where  $X_s(n\Delta f)$  is the Fourier transform at  $n\Delta f$  and

$$X_s(n\Delta f) = \int_0^{T_s} x_s(t) \exp(-i2\pi n\Delta f t) dt \quad (3.19)$$

The summation over the range  $[-M/2, +M/2]$  is sufficient as  $x_s(t)$  was filtered at  $f_d/2$  before the analog to digital conversion of the data. The

complex number  $(1/T_s) X_s(n\Delta f)$  describes the amplitude and the phase of the oscillator at the frequency  $n\Delta f$ .

The one-sided power spectrum  $G_x(f)$  was estimated as

$$G_x(n\Delta f) = \frac{2}{NT_s} \sum_{s=1}^N X_s^*(n\Delta f) X_s(n\Delta f) \quad (3.20)$$

where

$$n = 1, 2, \dots, M/2$$

and  $G_x(n\Delta f)$  is the density of the power in the frequency space; it indicates the contribution of the frequency interval  $[(n-1/2)\Delta f, (n+1/2)\Delta f]$  to the mean square value

$$\overline{(x(t) - \bar{x})^2} = \sum_{n=1}^{M/2} G_x(n\Delta f) \Delta f \quad (3.21)$$

The one-sided cross spectrum  $G_{xy}(n\Delta f)$  of a pair of functions  $x(t)$  and  $y(t)$  was estimated by calculating the Fourier transform of segments  $x_s(t)$  and  $y_s(t)$  and using the relationship

$$G_{xy}(n\Delta f) = \frac{2}{NT_s} \sum_{s=1}^N X_s^*(n\Delta f) Y_s(n\Delta f) \quad (3.22)$$

The cross spectrum is a complex-valued function and is represented as

$$G_{xy}(n\Delta f) = \text{Co}(n\Delta f) - i \text{Quad}(n\Delta f)$$

where  $\text{Co}(n\Delta f)$  is the cospectrum and  $\text{Quad}(n\Delta f)$  is the quadspectrum of the functions. The magnitude,  $G_{xy}(n\Delta f)$  is equal to the average product of the magnitudes  $|X_S(n\Delta f)|$  and  $|Y_S(n\Delta f)|$  and the phase is equal to the average phase difference of the two oscillators at  $(n\Delta f)$ . The covariance is equal to the sum over the cospectrum, or

$$\overline{(x(t) - \bar{x})(y(t) - \bar{y})} = \sum_{n=1}^{M/2} \text{Co}_{xy}(n\Delta f) \Delta f \quad (3.23)$$

The coherence function  $\gamma_{xy}^2$  is defined as

$$\gamma_{xy}^2(n\Delta f) = \frac{G_{xy}(n\Delta f)^2}{G_x(n\Delta f) G_y(n\Delta f)} \quad (3.24)$$

and is equal to unity, if the phase difference of the two oscillators  $X_S(n\Delta f)$  and  $Y_S(n\Delta f)$  is constant for all segments.

## IV. RESULTS

### A. Test Parameters

In accordance with the exploratory nature of this study, only a limited set of data was taken. The hull velocity  $V_0$ , or the derived frequency scale  $V_0/l_c$  ( $l_c$  = characteristic length of the ice sheet), was the main parameter that was varied in order to study the interaction of the ice forces experienced by the hull with its pitching motion. Two runs, or tests, were performed for each ice sheet: The first run involved the movement of the model hull at the relatively low velocity of 0.03 m/sec; and a second run was conducted with the model hull moving at a velocity of either 0.22, 0.3 or 0.6 m/sec. The flexural strengths of the ice sheets were chosen so that, for each hull velocity, runs were conducted for two values of  $\sigma_f$ . Ice-sheet thickness,  $h$ , was kept constant at about 0.023 m, with the exception of sheet G for which  $h = 0.032$  m. The physical parameters of the seven ice sheets used in the study are summarized in Table 5. The mean and standard deviations of draft, vertical acceleration, pitch and angular acceleration of the model hull are presented in Appendix 1. The buoyancy and inertia terms are additionally scaled according to (2.24) and (2.25) so that they are directly comparable. The scaling factors are listed in Table 6. Appendix 1 also gives the mutual covariance of the five quantities together with the correlation coefficients. The errors of measurement were discussed in the Section III.A.3. Table 4 is a summary of the error estimates.

In Appendix 2, examples of the time series of the measured quantities for every run, as scaled using (2.24) and (2.25), are presented. A scaling to prototype values was not the purpose of this study.



The resultant trends that were obtained for the temporal mean values of forces and moments experienced by the hull are discussed in Section B, while the time-dependent characteristics of the forces and moments are discussed in Section C.

## **B. Discussion of Mean Forces and Moments**

### **1. Calculated Data**

The measured\* temporal mean values of the ice-related resistance forces,  $\bar{F}_{xm}$  and  $\bar{F}_{zm}$  as well as the temporal mean values of the moment  $\bar{M}_{ym}$  are represented, in Figures 8a, b, c, as functions of hull velocity  $V_O$ . As was discussed earlier, it is assumed that  $\bar{F}_{xm}$  is approximately equal to the towing force  $\bar{T}_x$  (see (2.18)) ; that  $\bar{F}_{zm}$  is approximated by the buoyancy term  $\rho_w g A_w \bar{z}$  (see (2.19)); and that  $\bar{M}_y$  is approximated by the buoyancy term  $I_{yy} \rho_w g \bar{\theta}$  (equation 2.20).

The temporal mean horizontal ice force  $\bar{F}_{xm}$  generally increases with increasing hull speed. The measured mean buoyancy force  $\bar{F}_{zm}$  increases with hull speed up to  $V_O = 0.3$  m/sec; and subsequently decreases toward  $V_O = 0.6$  m/sec. For an ice-sheet thickness of 0.023 in, the moment  $\bar{M}_{ym}$  is constant for hull speeds up to a value of 0.3 m/sec and thereafter increases for speeds upto a value,  $V_O = 0.6$  m/sec.

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\*Measured quantities have the subscript m.

## 2. Comparison with Data Obtained by Arctec Inc.

Extensive model tests of the USCGC "Polar Star" were conducted in 1976 by Arctec Inc. of Maryland USA (Lecourt and Deslauriers, 1976). For these tests, Arctec used saline ice (NaCl solution), instead of urea ice, with flexural strength ranging from 7 to 12 kPa.

Values of the resistance force,  $\bar{F}_{xm}$  as were obtained in the present study are compared in Figure 9 with those obtained during the tests conducted by Arctec. The IIHR data are in good agreement with those Arctec data that are associated with a coefficient of friction, between ice and hull, of  $F_{ID} = 0.124$ . The Arctec data for  $F_{ID} = 0.478$ , which was associated with the initial surface finish of the model hull (prior to its use by the IIHR) are consistently higher than the IIHR data. During the present study, no measurement of  $F_{ID}$  was made with urea ice.

## 3. Effects of the Depression of the Water Level in the Bow Region

The trends of  $\bar{F}_{xm}$ ,  $\bar{F}_{zm}$  and  $\bar{M}_{ym}$ , that are shown in Figure 8, can be interpreted by considering the effect that the depression of an ice sheet has on the distribution of hydrostatic pressure that is exerted against the hull (as discussed in Section II.B). Because little information is available on these trends, it is of interest to get at least approximate estimates.

At the relatively low value of hull velocity  $V_0 = 0.03$  m/sec, the loaded ice sheet was flexed down below the water level and became flooded with water. The photograph Figure 10a illustrates the flooding of such a semicircular area which had a diameter of about twice the beam of the hull. For moderate hull velocities,  $V_0 = 0.22$  m/sec, this area did not flood (Figure 10b), and at  $V_0 = 0.6$  m/sec, not only did flooding not occur but a small air

gap was visible between the deflected surface of ice sheet and the hull's bow (Figure 10c). It can be concluded from these observations that, at  $V_0 = 0.03$  m/sec, the distribution of the hydrostatic pressure against the hull is not affected by the depression of the ice sheet and that the analysis as is embodied in (2.7) through (2.12) can be used to estimate the actual ice forces exerted against the hull. The influence of water surface depression due to the downward flexing of the ice sheet becomes progressively significant at higher velocities of hull motion. This result is supported by the observation that at  $V_0 = 0.03$  m/sec, the measured vertical ice force  $\bar{F}_{zm}$  correlates well with the range given by the two contributions in (2.23),  $F_z = A_w (\rho_w - \rho_i) gh + \sigma_f h^2$  (Figure 11). The second term is time dependent and contributes only with its temporal mean to  $\bar{F}_z$ . The coefficients  $\alpha_2$  and  $\alpha_3$  in (2.23) were set equal to unity for this comparison.

In order to analyze the effect of ice sheet depression for higher velocities of hull motion, estimates of the ice force  $\bar{F}_z$  and its moment  $\bar{M}_y$  are required in order to evaluate  $\Delta B'_z$  and  $\Delta M'_y$  (see (2.18) and (2.19)); where

$$\bar{F}_z = \bar{F}_{zm} + \Delta B'_z \approx \bar{F}_{zm} + A_1 \rho_w \alpha_1 z$$

and

$$\bar{M}_y = \bar{M}_{ym} + \Delta M'_y = \bar{M}_y + \Delta B'_z L$$

The estimates can be derived from the ratios  $\alpha_1 = \bar{F}_{zm}/\bar{F}_{xm}$  and  $L_I = \bar{M}_{ym}/\bar{F}_{zm}$  which are plotted in Figures 12a, b. For relatively low hull velocities, values of  $\alpha_1$  show little scatter and a mean value of  $\alpha_1$ , for the ice sheets tested, can be evaluated as

$$\bar{\alpha}_1(V_O = 0.03 \text{ m/sec}) = 1.5 \pm 0.3 \quad (4.1)$$

It is evident from Figure 12 that  $\alpha_1$  decreased with increasing hull velocity for all the ice sheets that were tested. However,  $\alpha_1$  is within the range of values given in (4.1), except when  $V = 0.6 \text{ m/sec}$ , for which the value of  $\alpha_1$  dropped to about one half (sheet F) and one third (sheet E) of its value given in (4.1).

As stated in Section III.2.C, it can be assumed, as an approximation, that  $\alpha_1$  is constant provided that the distribution of the ice pressure and the motion of the broken ice is similar. If it is further assumed that  $\alpha_1 = 1.5$  when  $V_O = 0.6 \text{ m/sec}$  and that the characteristic area  $A_1$  of the depressed water level is of the order of 10-percent of  $A_w$ , it follows from (2.14) and (2.19) that

$$\Delta B'_z \approx 1.5 \bar{F}_{xm} - \bar{F}_{zm} \quad (4.2)$$

and

$$\Delta z = \frac{1.5 \bar{F}_{xm} - \bar{F}_{zm}}{0.1 A_w \rho_w g} \quad (4.3)$$

Estimated values of  $\Delta B'_z$  and  $\Delta z$  are listed in Table 7. For ice sheets E and F, the estimated value of  $\Delta z$  is 0.025 m. This value seems to be the right order of magnitude. It is, however, a quandary as to how the ice sheet can transmit the total vertical force  $F_z$ , which considerably exceeds its bearing capacity for a stationary load.

Calculation of the buoyancy force increment  $\Delta B'_x$ , using (2.16) with  $\Delta z = 0.25\text{m}$ , indicates it to be small compared to the measured resistance, or the towing force,  $T_x$ . The approximation  $\bar{F}_x = \bar{T}_x$  is therefore a valid assumption for the given margin of experimental errors.

The ratio  $\bar{M}_y/\bar{F}_z$  is an estimate of the moment arm  $L_I$  of the resultant ice force acting against the hull. When  $V_O = 0.03\text{ m/sec}$ , the mean value of  $L_I$  was

$$\bar{L}_I = 0.73 \pm 0.08\text{ m} \quad (4.5)$$

For moderate hull velocities,  $L_I$  marginally decreased to  $0.63\text{ m}$  with a root-mean square value of  $0.13\text{ m}$ . When  $V_O = 0.6\text{ m}$ ,  $L_I$  increased to  $1.04\text{m}$  (sheet F) and  $1.54\text{ m}$  (sheet E). This estimate for  $L_I$  is in contradiction to the requirement that the line of action of the ice forces must lie within the hull. In other words, this result supports the argument that  $\bar{F}_{zm}$  must underestimate the true value of the vertical ice forces acting against the hull.

The moment increment  $\Delta M'_y$  can be estimated using an argument similar to that used for estimating the force increment  $\Delta B'_z$ : If  $L_I = 0.73\text{m}$  is an adequate estimate of the moment arm for all hull velocities, then

$$\bar{M}_y \approx L_I \bar{\alpha}_1 \bar{F}_x$$

and

$$\Delta M'_y = L_I \bar{\alpha}_1 \bar{F}_x - \bar{M}_{ym} \quad (4.6)$$

The moment arm,  $L$ , of the buoyancy force increment  $\Delta B'_z$  is

$$L = \frac{L_I \bar{\alpha}_1 \bar{F}_x - \bar{M}_{ym}}{\Delta B'_z} \quad (4.7)$$

For ice sheets E and F,  $L = 0.35\text{m}$  and  $0.41\text{ m}$ , respectively. However, these estimates are too approximate to place the depression of the ice sheet in the bow region.

The results summarized in Table 7 indicate that the depression of the water level at the bow region of the hull depressing an ice sheet plays an important role in the generation of ice forces. It leads to an increase in the resistance to motion that is experienced by the hull. There is a need for more detailed experiments to investigate the influence of water level depression, at the bow region, on the dynamic response of an ice-breaker hull to ice breaking.

### C. Temporal Variation of Forces and Moments

In section II.D, the motion for the heaving and pitching motions of a hull were analyzed as forced oscillations and the following relationships were proposed:

$$m_V \ddot{z} + c_1 \dot{z} + \rho_w g A_w z = F_z(t) \quad (2.24)$$

$$I_V \ddot{\theta} + c_2 \dot{\theta} + \rho_w g I_{yy} \theta = M_y(t)$$

Inertia	Damping	Buoyancy	Forcing function	(2.25)
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The vertical ice force  $F_z(t)$  and the moment  $M_y(t)$  act as forcing functions which are in equilibrium with the inertia forces and the buoyancy forces. The inertia and buoyancy forces were measured during each of the

tests. However, the damping terms could not be isolated in the experiment. In particular, friction in the ball bushing support for the model hull caused a fluctuating vertical force of unknown size to act through the center of gravity of the hull and to damp fluctuations of  $F(t)$ . Errors due to this damping effect have considerably influenced the vertical forces that were recorded during experimentation.

### 1. Amplitudes of Forces and Moments

It is useful to compare peak-to-peak values of the fluctuations of the inertia and the buoyancy forces with an estimate of the ice forces that were needed to flexurally fail the ice sheet. The load cycle associated with the increase and subsequent relaxation of the ice forces was considered to be the source of the temporal fluctuations of the forcing functions  $F_z(t)$  and  $M_y(t)$  acting on the hull. In Figures 13a and b, typical maxima and minima of peak-to-peak values of  $\rho_w g A_w \ddot{z}(t)$  and  $m_V \ddot{z}(t)$ , respectively are scaled with  $\sigma_f h^2$ , and are plotted against the frequency parameter  $V_0/l_c$ . In Figures 13c and d, typical maxima and minima of peak-to-peak values of  $\rho_w g I_{yy} \ddot{\theta}(t)$  and  $I_V \ddot{\theta}(t)$ , respectively, are scaled with  $\rho_f h^2 L_I$  and are plotted versus  $V_0/l_c$ . The moment  $L_I$  of the ice force acting against the hull is taken to be  $L_I = 0.73$  m.

The bearing capacity of an infinite ice sheet sustaining a circular load of finite diameter is reported to be a factor of 1.5 to 2 times  $\sigma_f h^2$ . For a semiinfinite ice sheet, the factor is 0.4 to 0.6 (Michel 1978). The buoyancy and inertia forces that were measured in this study were 180 degrees out of phase with each other. Consequently these forces partly compensated one another, especially at the resonant frequency (2.32). The observed minimal peak-to-peak values appear to be good indicators for the fracture load and the



maximal peak-to-peak values indicate the presence of a strong resonance. From this point of view, the fracture load, as is shown in Figure 13, has a magnitude between 0.4 and 0.8 times  $\sigma_f h^2$ , for the model hull. This result is in agreement with the fracture load of a semiinfinite ice sheet. Strong resonance in the pitching motion of the hull was recorded for values of  $V_0/l_c$  between 1 and 2. At high velocities ( $V_0/l_c = 3.4$  and  $4.2$ ) higher peak-to-peak values were observed. This result can possibly be attributed to the effects of the inertia forces of the ice floes and the water underneath the ice.

## 2. Spectral Distributions of Forces and Moments

In section II.D it was assumed that the period between the formation of consecutive cracks in the ice sheet has a maximum value of  $l_c/V_0$ , where  $l_c$  is the characteristic length of the ice sheet and  $V_0$  is the speed of the hull. If the rate of increase of the ice force is not too great, the spectrum of the forcing function has a peak value at the frequency  $v_0/l_c$ .

The response of the hull was determined by calculating the power spectra, the cross spectra and the coherence functions for the temporal records of the five measured variables of hull motion--i.e., draft, pitch, vertical and angular accelerations and towing force. The power spectra, cross spectra and coherence functions for the measured variables are summarized in Appendix C.

The bandwidth of the data channels are listed in Table 3. The upper frequency limit of  $F_x$ , given by the filter frequency of 3 Hz, appeared to be inadequate, especially at higher velocities (sheets E,F) and for sheets of thicker ice (sheet G). The draft and pitch motions of the hull were not filtered. The decline of their spectra is therefore given by the motion of the hull. The accelerations were filtered at a frequency of 6 Hz. The low

frequencies that were recorded for the vertical acceleration  $\ddot{z}(t)$  are due to rolling motion of the hull.

Examples of the time histories of forces and moments are given in Appendix 2. From these examples, the relative size of the terms in (2.24) and (2.25) can be qualitatively estimated. The power spectra (Figure C1) give, in a more quantitative way, the average contribution of each frequency interval to the mean square of the fluctuation and enable a comparison to be made of the strength of the forces and moments in the frequency space. The cross-spectra and coherence functions of the pairs  $z(t)$  and  $\ddot{z}(t)$ ,  $\theta(t)$  and  $\ddot{\theta}(t)$  (see Figure C2),  $F_x(t)$  and  $z(t)$ ,  $F_x(t)$  and  $\ddot{z}(t)$ , (see Figure C3),  $F_x(t)$  and  $\theta(t)$ , and  $F_x(t)$  and  $\ddot{\theta}(t)$  (see Figure C4) provide information on the frequency range where both quantities have simultaneously strong spectral lines, and information on the average phase relation. The cross spectra and the coherence functions also facilitate the checking of the data for consistency. The theoretical coherence functions of  $z$  and  $\ddot{z}$ , as well as  $\theta$  and  $\ddot{\theta}$ , are unity. Any deviation of the actual values from the theoretical values must be due either to unwanted filtering, to noise if the signals are small, to error of measurements or to a violation of the basic assumptions which lead to the formulation of (2.23) and (2.24). The coherence function of  $\theta(t)$  and  $\ddot{\theta}(t)$  (pitch with angular acceleration) is close to unity in a frequency band around the resonant frequency  $f_0 = 1\text{Hz}$ . The coherence function decreases to zero for higher frequencies of pitching motion. The relationship between  $z(t)$  and  $\ddot{z}(t)$  (draft and vertical acceleration) are less coherent if the fluctuations of the forces are small. This may be due to friction in the ball bushings as was mentioned before. The cross spectra of  $\theta(t)$  and  $\ddot{\theta}(t)$  show the expected behavior. The cospectrum has normally a

negative peak at the resonance frequency and the quadspectrum is close to zero. This result agrees with the fact that  $I_V \ddot{\theta}$  is 180 degrees out of phase with  $\rho_w g I_{yy} \theta$ .

In Appendix C, one set of power spectra and the three sets of cross spectra and coherence function are organized in the following way: Figure (a) of each set gives examples of three low velocity runs; Figure (b) collects the runs at medium velocity ( $V_O = 0.22$  m/sec and  $V_O = 0.30$  m/sec); Figure (c) shows the high velocity runs ( $V_O = 0.6$  m/sec); and Figure (d) gives the results for the two runs of sheet G, which was distinguished by its greater ice thickness.

### 3. Classification of Tests According to Frequency Response of the Hull

The tests can be classified according to the frequency response of the hull to the forcing functions exerted against it during the process of ice breaking. As is described in Section II.D, the parameter  $V_O/f_{OLC}$  can be used as a measure of the dynamic, or frequency, response of the hull to ice forces. It was found for the model hull that the natural frequencies of heaving and pitching are sufficiently equivalent that the two resonances cannot be separated. Values of  $V_O/f_{OLC}$  for the tests are given in Table 5.

For the relatively low hull velocity of  $V_O = 0.03$  m/sec, when  $V_O/f_{OLC}$  is significantly less than unity, the acceleration terms are small and the buoyancy terms are in equilibrium with the forcing function due to ice forces. The power spectra of the inertia terms are zero and the power spectra of pitch  $\theta$  and  $F_x$  are nonzero only at the lowest frequencies (Figures C1a,d). These two quantities show there are also some positive values in the co-spectrum, which means that they are in phase (Figures C4a through d).

An average distance,  $\Delta x$ , between consecutive cracks in the ice sheets can be estimated from the fluctuations in the temporal records of pitch angle  $\theta$ . Values of the measured frequency of the ice force cycle,  $f = V_O/\Delta x$ , are compared with  $V_O/l_C$  in Figure 14. In Figure 15 the same data is used to correlate the distance  $x$  directly with  $l_C$ .

At hull velocities for which  $V_O/f_O l_C$  is in the range of 1 to 2, buoyancy and inertia terms are of equal magnitude and partly compensate one another, as is indicated in (2.33). The cross-spectra and power spectra, as shown in Figure C1-4b, depict the resonance effect, especially for the pitch and angular acceleration of the hull  $\theta(t)$  and  $\ddot{\theta}(t)$ . For sheet G the resonance is also evident for the vertical motion of the hull,  $z$  and  $\ddot{z}$ . As can be seen from the positive resp. negative cospectrum given in Figures C4b through d (sheet G),  $F_x$  is in phase with  $\theta(t)$  and  $z(t)$  at the resonant frequency  $f_O$  and out of phase with  $\ddot{\theta}(t)$ .

For high hull velocities, with  $V_O/f_O l_C$  approximately equal to 4, the power spectra of the buoyancy terms decrease toward higher frequencies. Ice sheets are broken predominantly by inertia forces, which show strong lines in the spectra at  $V_O/l_C$  (Figure C1c). For sheet F, when  $V_O = 0.6$  m/sec, there is a peak in the cospectrum of  $z$  and  $\ddot{z}$  as well as  $\theta$  and  $\ddot{\theta}$  at the frequency  $f = 1.3$  Hz. This indicates the presence of transients (see (2.32) and (2.33)). There is also a dominant peak in the power spectra of  $F_x$  at the frequency  $f = 1.6$  Hz (Figure C1c) which is also clearly visible in the time histories (Figure B). This peak is also apparent in the cross spectra of  $F_x$  and  $\ddot{z}(t)$  and  $F_x$  and  $\ddot{\theta}(t)$  (Figures C3c and C4c). The phase angle is not clearly defined by the ratio of the cospectrum and the quad-spectrum because at this frequency the phase error of the data acquisition is about 10 percent

of  $2\pi$ . The distance travelled by the hull during the period  $T = 0.62$  sec, corresponding to the dominant frequency of ice forces,  $f = 1.6$  Hz, is equal 0.36 m. This length, which is 2 times  $l_c$ , is an indication that these fluctuation of  $F_x$  may be produced by an interaction effect of the pitch motion and the breaking of the ice sheet. As was stated in Section II.D, such an interaction effect is not included in the present analytical model of forced oscillation of an ice-breaker hull. The accuracy of the spectra is marginal-limited at high velocities of hull motion, because the observation time and data acquisition is restricted by the length of the towing tank.

The peaks in the power spectra can be used to estimate the recorded frequency  $f_m$  of the forcing function and to estimate the distance,  $\Delta x$ , between consecutive cracks formed in the ice sheet as the hull moved through it. For the high velocity runs, the peaks at the highest frequency were chosen. Figure 14 shows the correlation of  $f_m$  with the frequency scale  $V_0/l_c$  and Figure 15 shows the same information as a correlation of  $V_0/f_m$  with  $l_c$ . The good correlation that is evident in Figures 14 and 15 is a confirmation that the average value of the initial breaking length,  $\Delta x$ , is adequately estimated using the characteristic length of the ice sheet,  $l_c$ . This result, nonetheless, is somewhat contradicted by the observation that, at relatively high hull velocities, the mean size of the ice floes in the wake of the hull is significantly smaller than  $l_c$ . Figure 16 shows typical sizes of these ice rubble, after a run at a relatively low velocity,  $V_0 = 0.03$  m/sec (Figure 16a), and after a run at a relatively high velocity,  $V_0 = 0.60$  m/sec (Figure 16b). However subsequent fracture of the ice floes at high velocity during passage of the hull may explain this contradiction.



## VI. CONCLUSIONS

Based on experimentation involving a 1:48-scale model of the USCGC ice-breaker ship "Polar Star" moving with constant velocity through level ice sheets, the following principal conclusions were drawn on the dynamic response of an ice-breaker hull to ice breaking:

1. The dynamic behavior of an ice-breaker hull breaking ice can be described in terms of a forced oscillation and classified according to the parameter  $V_0/f_0 l_c$ , where  $V_0$  = the hull's velocity,  $f_0$  = the natural frequency of pitching (and heaving) motion of the hull.  $l_c$  = the characteristic length of the ice sheet, relates directly to the distance between consecutive cracks formed circumferentially around the bow of an ice-breaker hull.

The flexural failure of an ice sheet by a hull causes a quasiperiodic forcing function of frequency  $V_0/l_c$  to be exerted against the hull. The amplitude of the forcing function was estimated to be in the range of 0.4 to  $0.8 \sigma_f h^2$ .

For relatively low hull velocities,  $V_0/f_0 l_c \ll 1$ , the buoyancy terms in the equation of motion are approximately equivalent to the ice forces, or the forcing function, exerted on the hull during ice breaking. When an ice sheet is broken by a hull moving at relatively high velocity,  $V_0/f_0 l_c \gg 1$ , the inertia forces of the hull are principally responsible for ice breaking. When a hull is moving at a velocity such that  $V_0/f_0 l_c = 1$ , both buoyancy and inertia forces of the hull are large compared to ice forces. These forces partly compensate one another, because they are 180-degrees out of phase.

2. For relatively high cruise velocities, the hydrostatic pressure distribution acting against the hull may be reduced because the depressed portion of the ice sheet at the hull's bow may no longer be flooded with water.

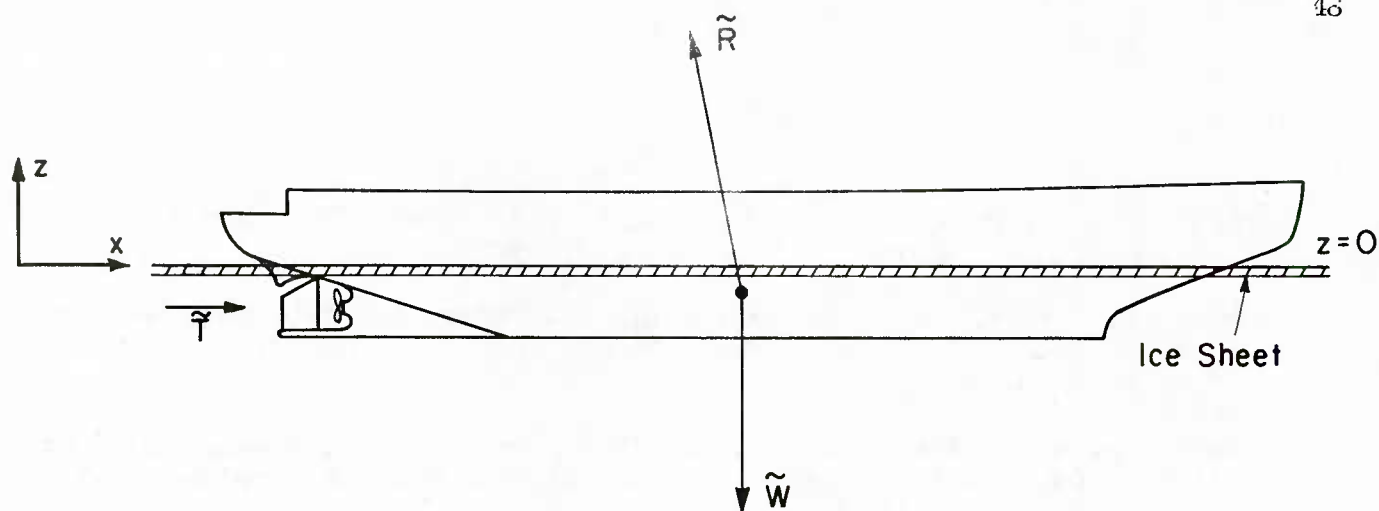
The distance that the leading edge of the ice sheet was depressed and the effect on the hulls motion were estimated using the assumption that the ratio of vertical and horizontal ice forces is a constant value of about 1.5. The depression of the ice sheet, and the water level was estimated to be of the order of 0.01 m for a model hull moving at a velocity of 0.6 m/sec. This depression of water level was estimated to produce about one half of the reduction of the hulls buoyancy necessary to load the ice sheet.

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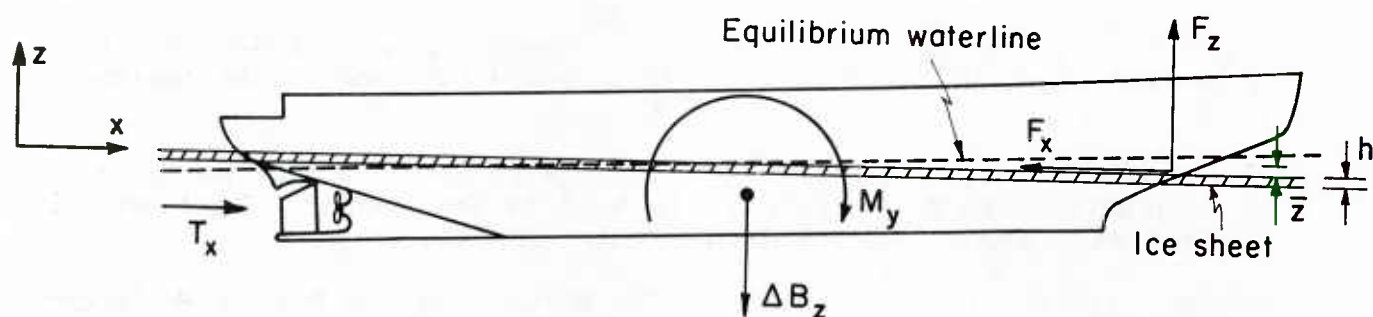
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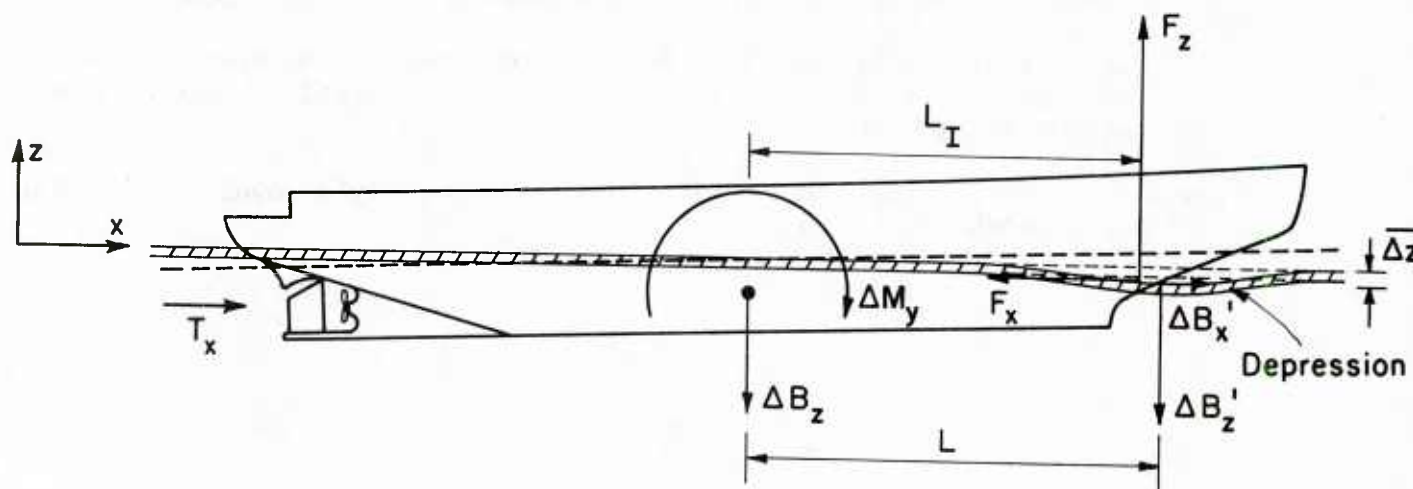
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(a) Total forces

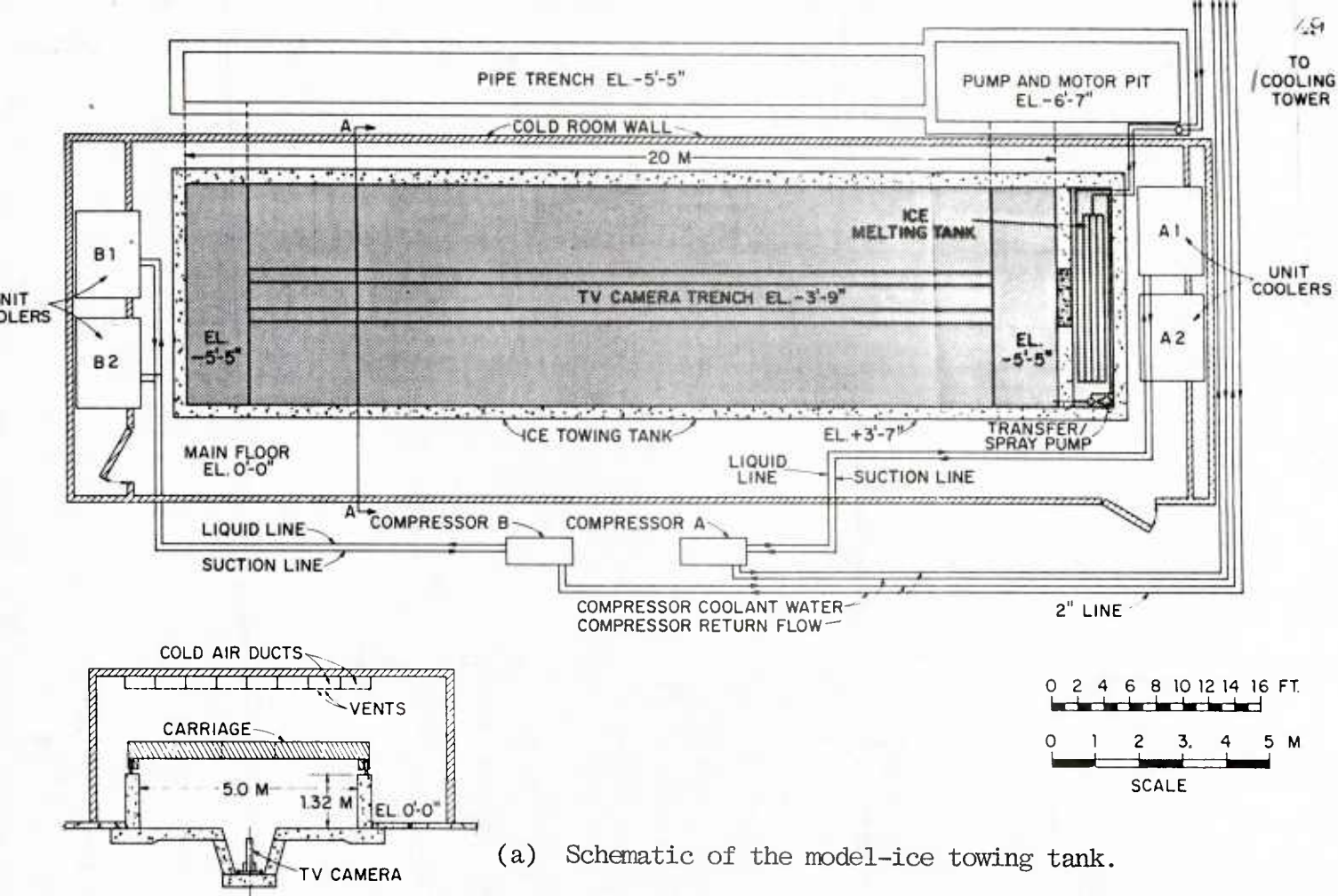


(b) Forces and moment due to ice breaking



(c) Influence of ice-sheet and water level depression, at the bow, on the loading of a hull.

Figure 1. Definition sketch of forces acting on an ice-breaker hull.



(a) Schematic of the model-ice towing tank.

SECTION A-A



(b) Towing tank and motorized carriage

Figure 2. The IIHR model-ice towing tank

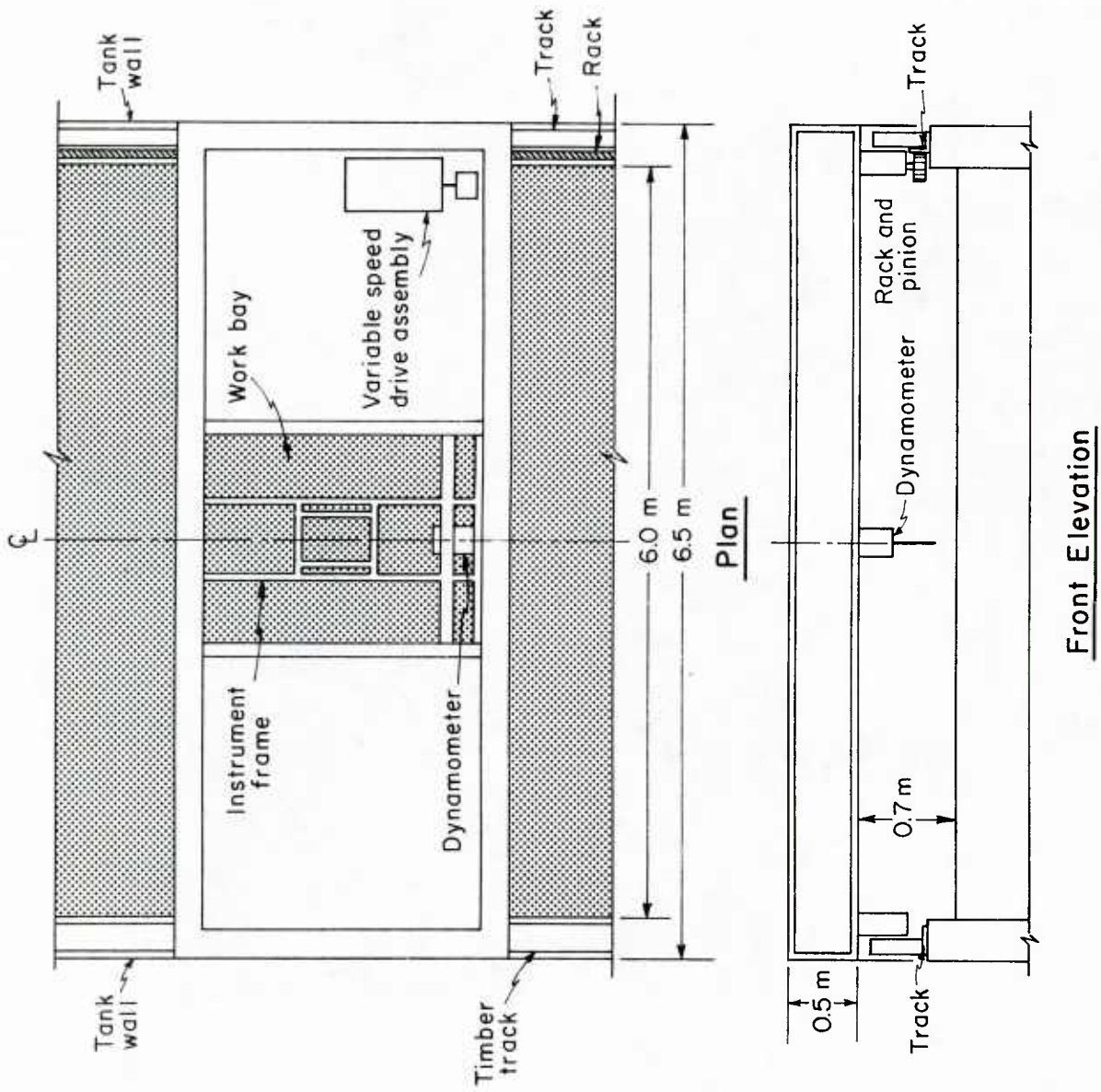


Figure 3. Sketch of the motorized carriage



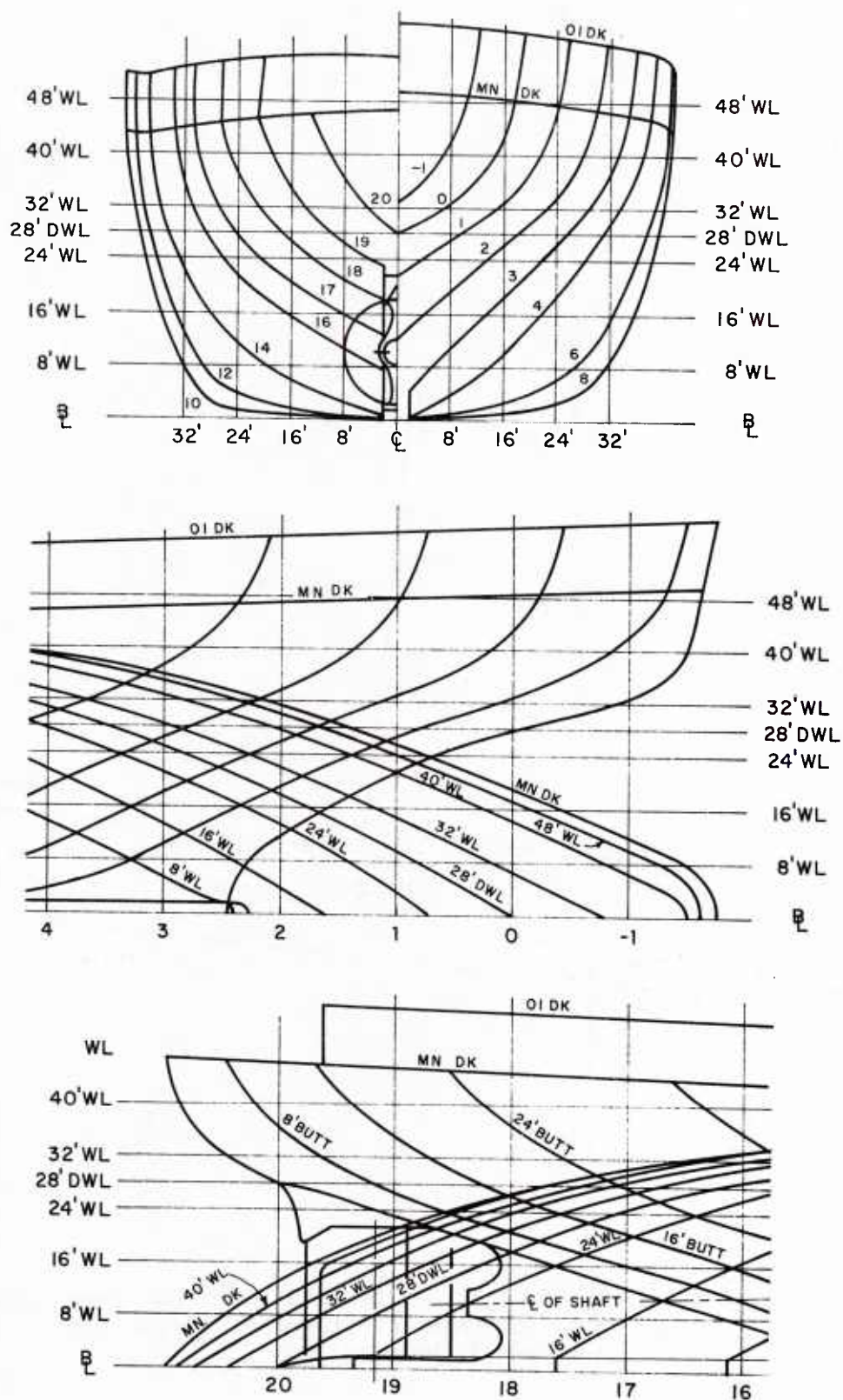


Figure 4. Abbreviated lines of the USGSC ice-breaker ship 'Polar Star'

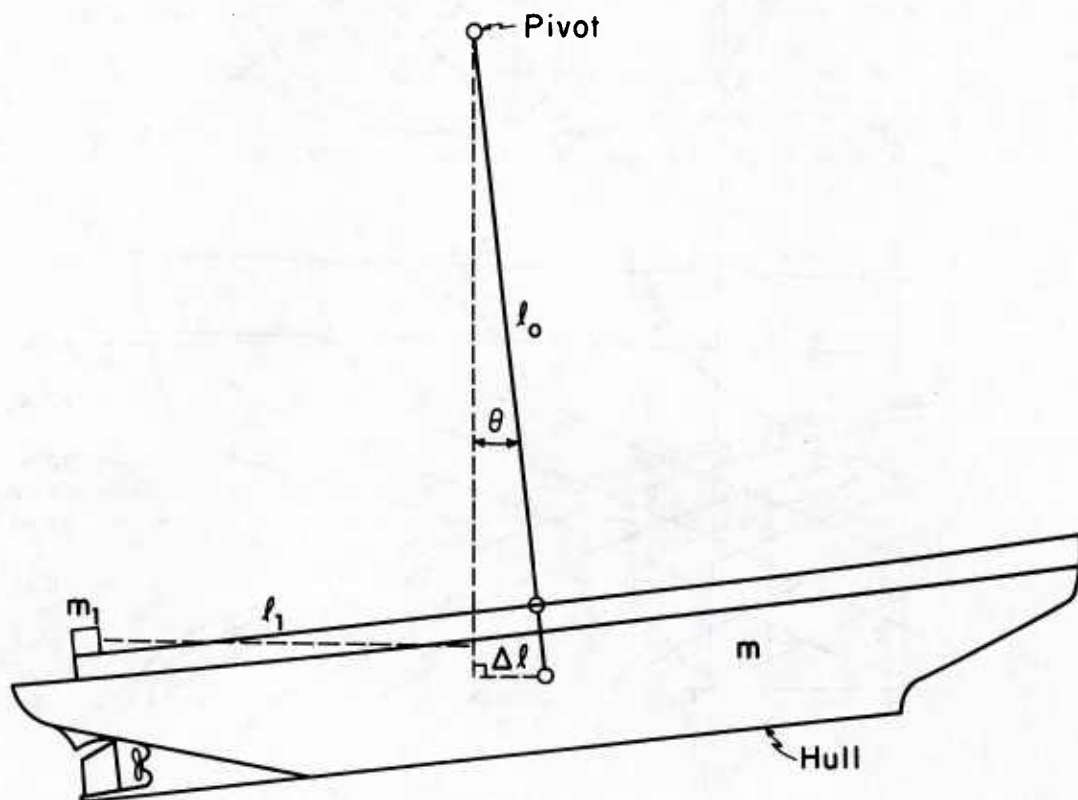


Figure 5. Definition sketch of the pendulum arrangement that was used to determine the mass moment of inertia of the model hull.

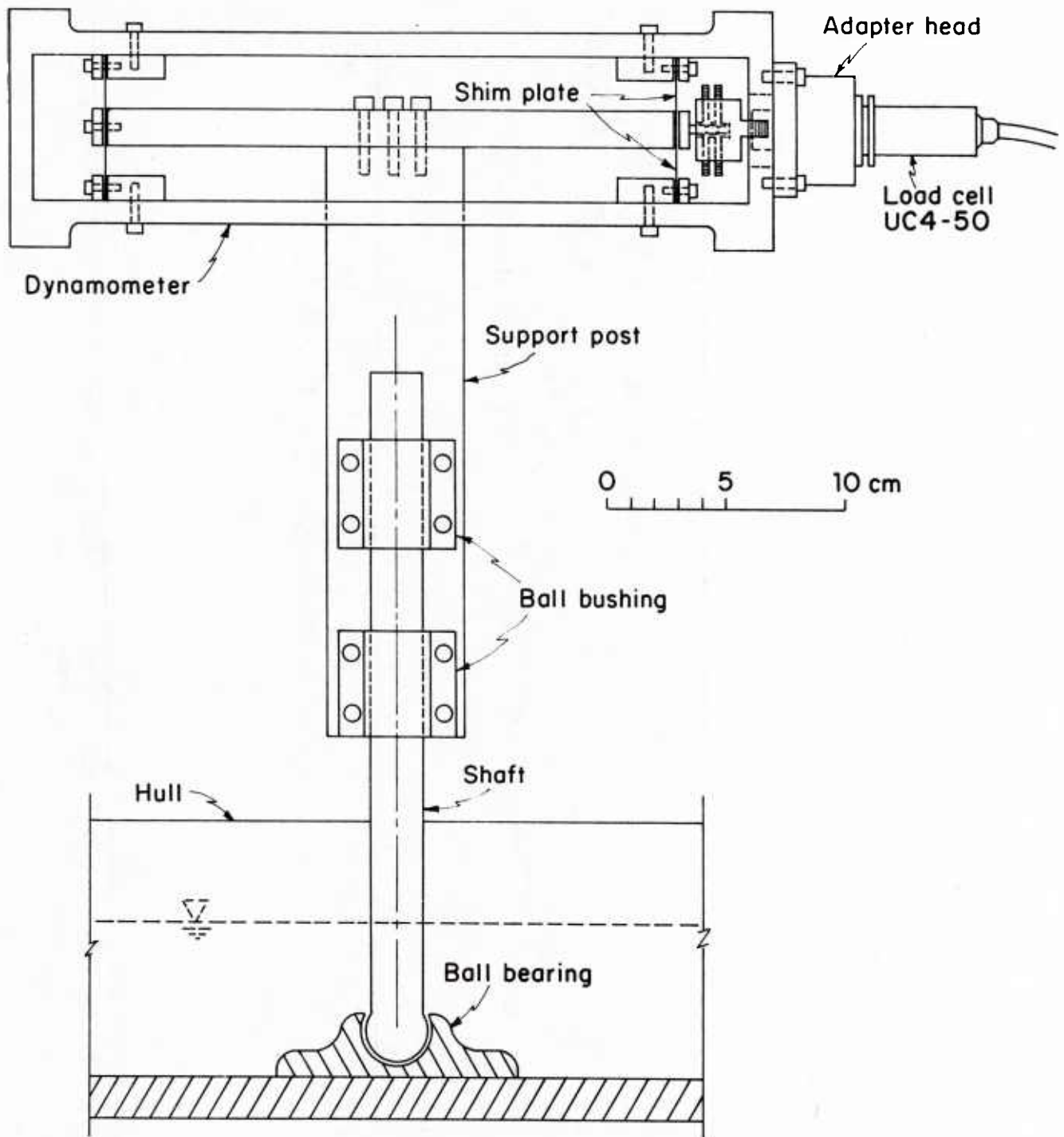


Figure 6. Details of the dynamometer that was used for measuring towing force.



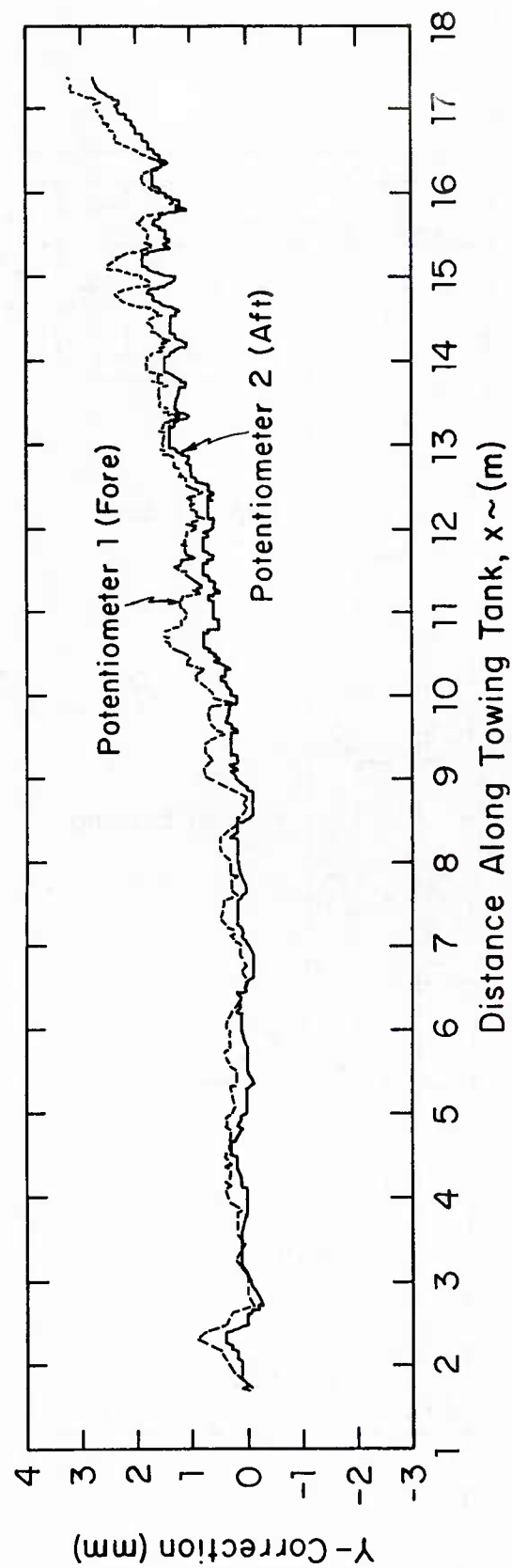


Figure 7. Relative height,  $H_R(x)$ , or vertical correction for position along the tracks of the motorized carriage.

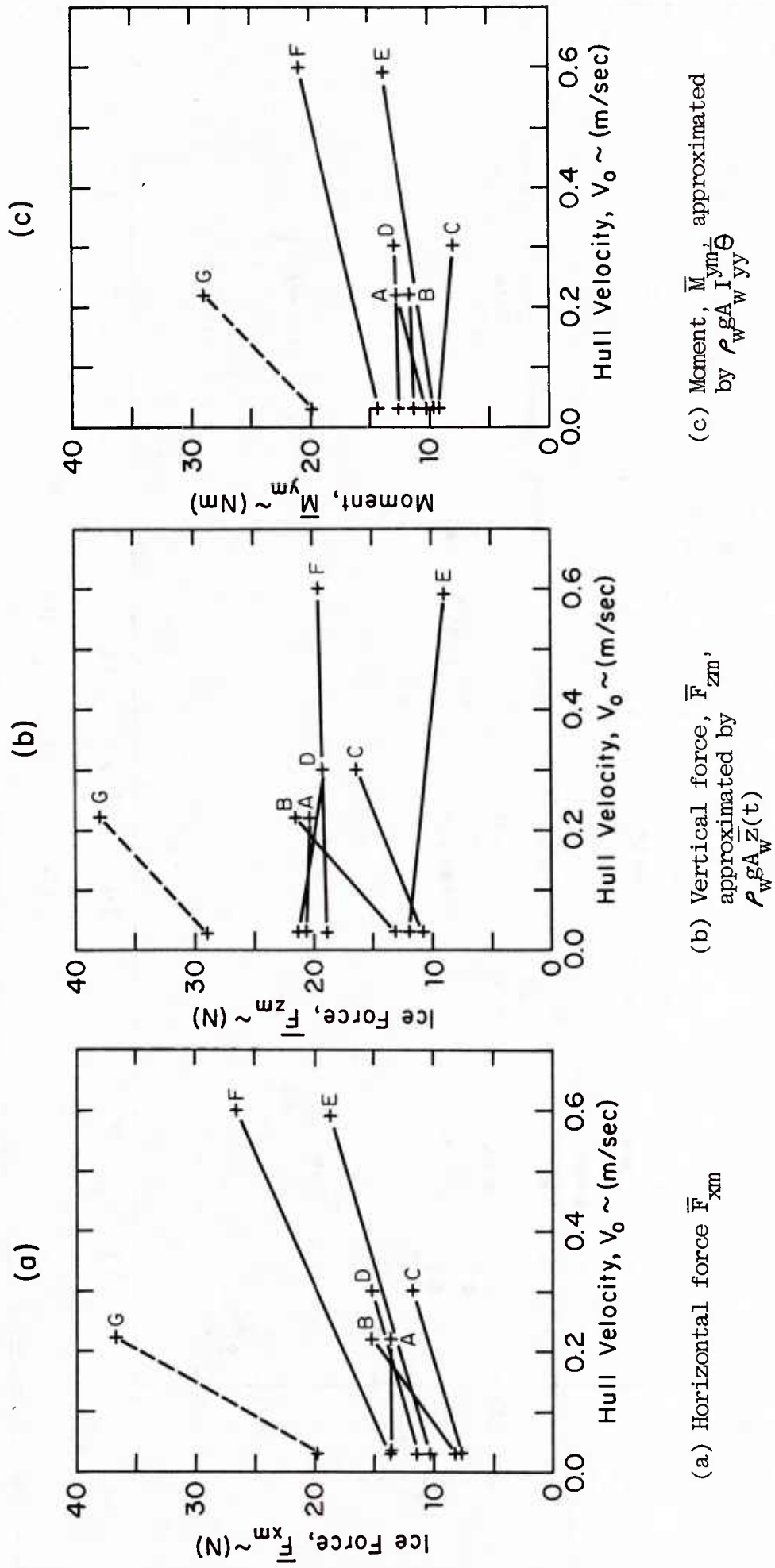


Figure 8. Temporal mean values of measured forces and moments acting against the hull.

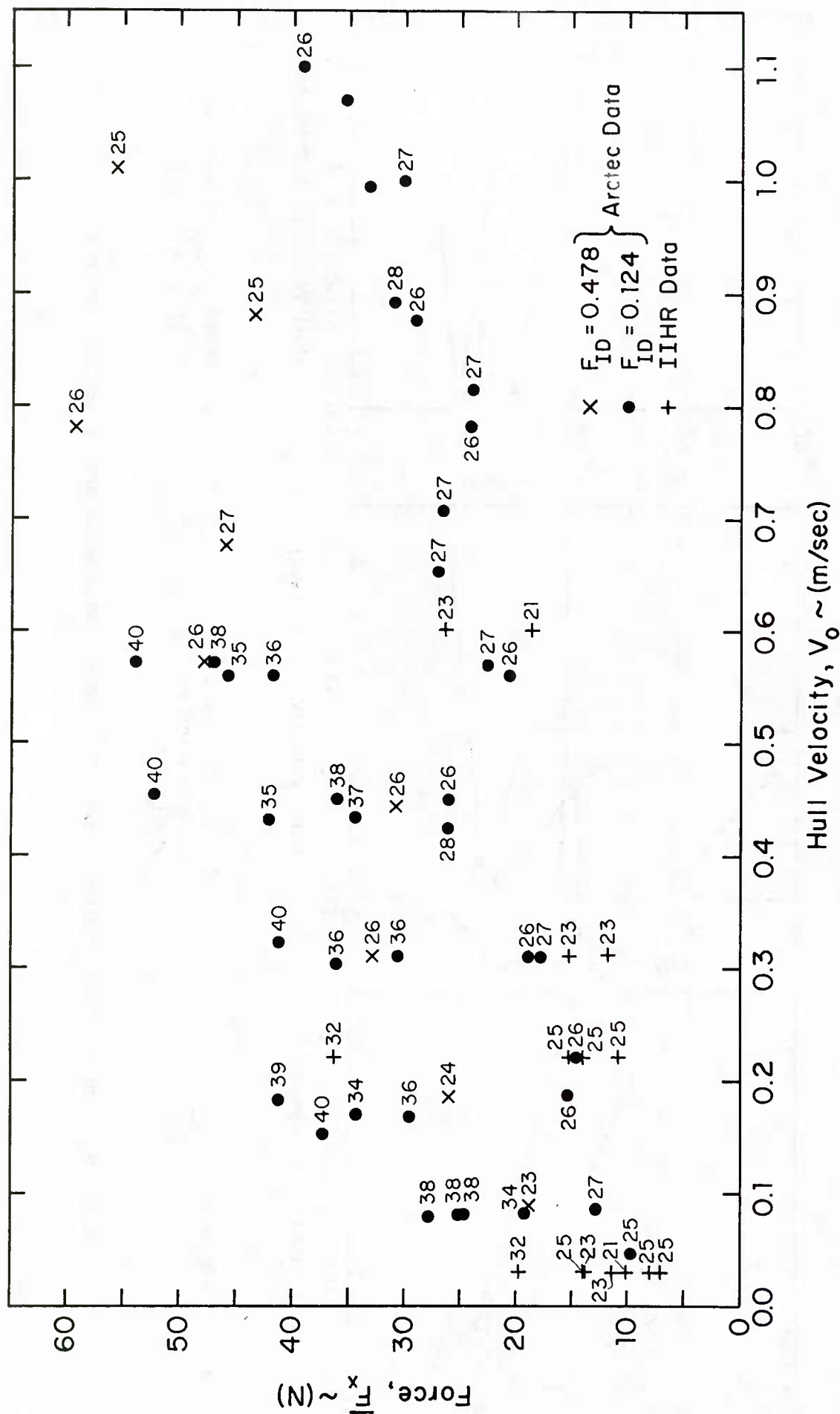
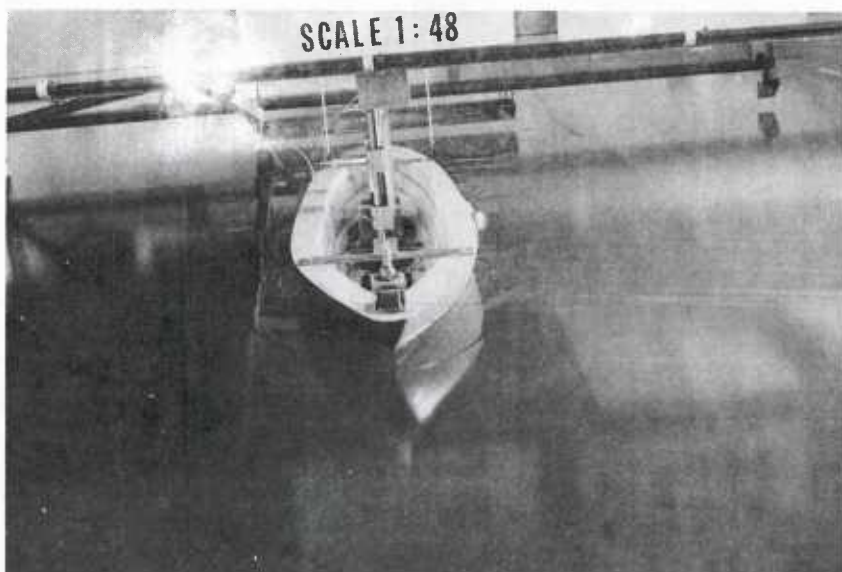
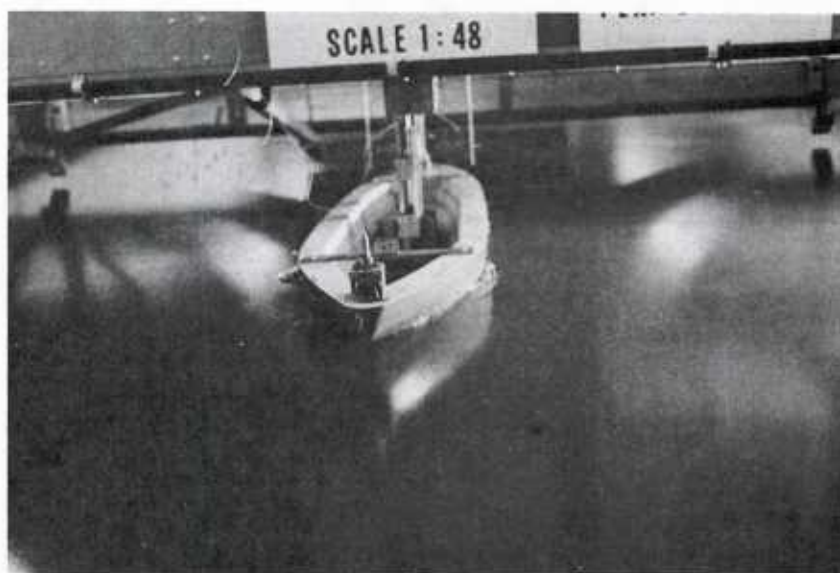


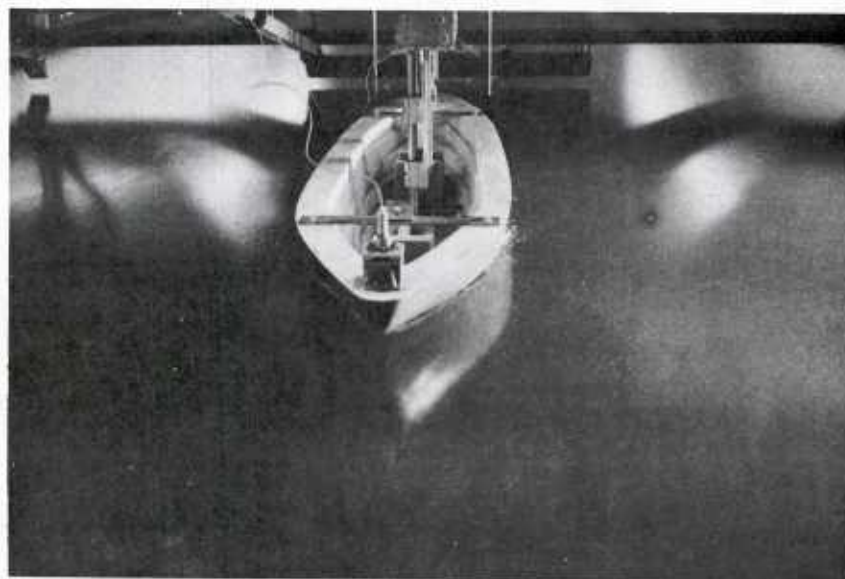
Figure 9. Comparison of  $\bar{F}_{xm}$  with the results of the model tests conducted by Arctec Inc. The number on each plot point indicates ice sheet thickness in mm.



- (a) For a low speed  
( $V_0 = 0.03$  m/s), the  
depression is flooded



- (b) For a medium speed  
( $V_0 = 0.22$  m/s), the  
depressed area is dry



- (c) For a high speed  
( $V_0 = 0.60$  m/s), an  
air gap was visible  
at the bow

Figure 10. Depression of the  
ice sheet at the bow of the  
hull.

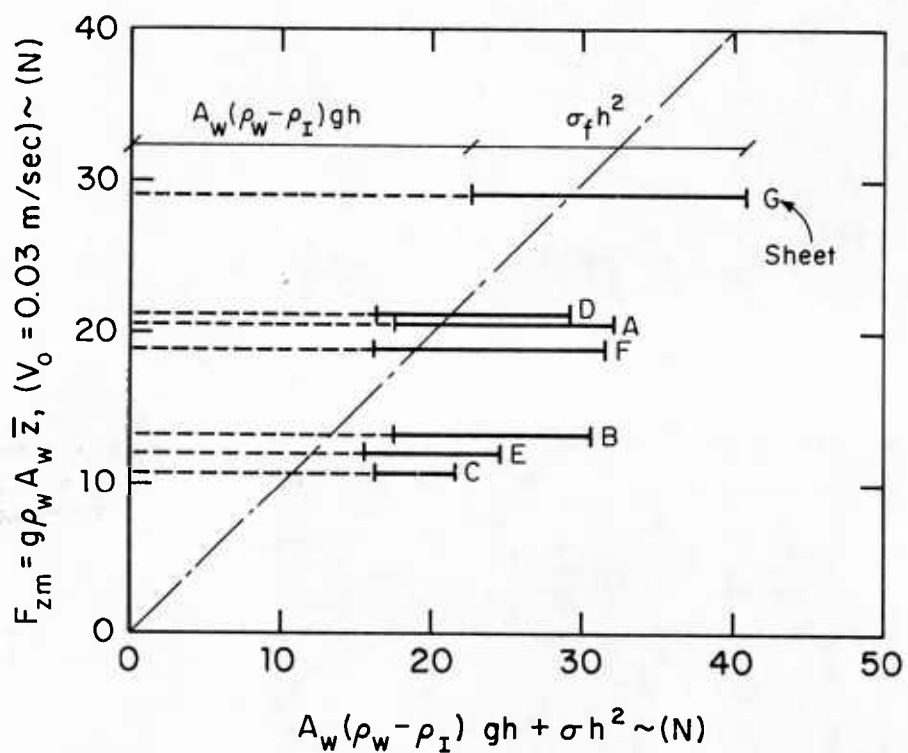
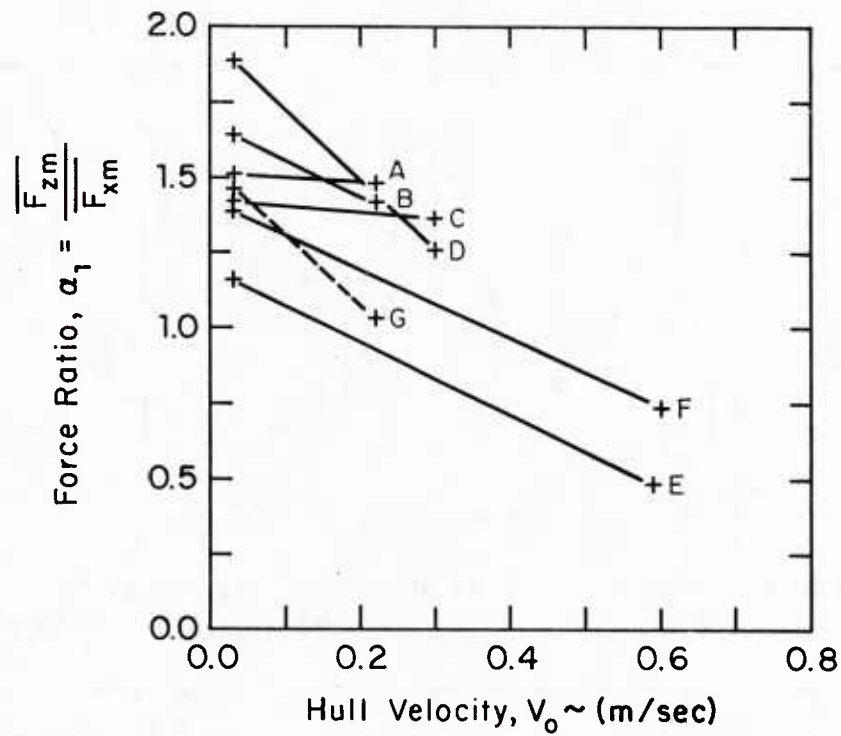
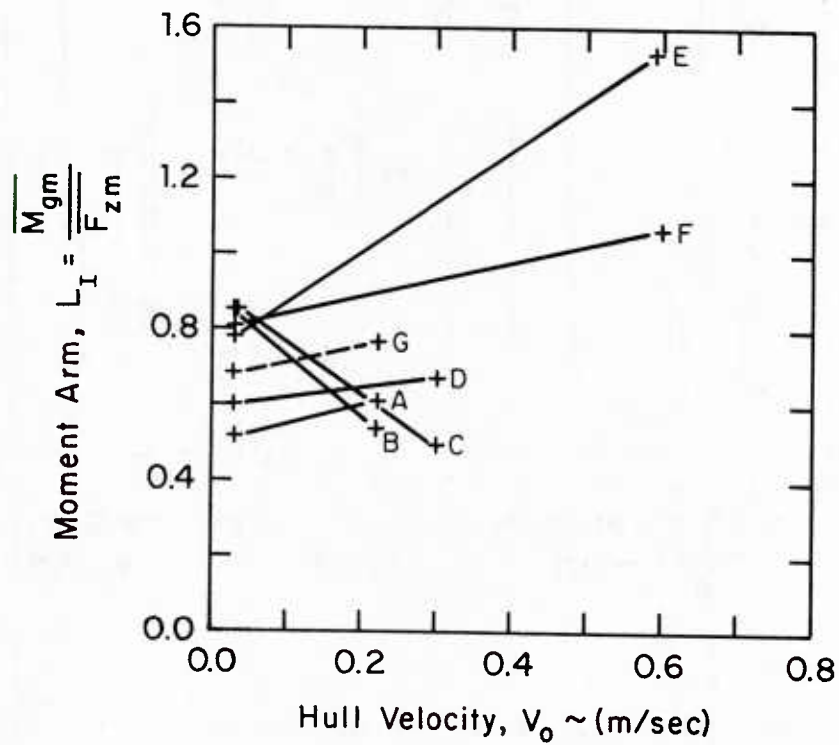


Figure 11. Comparison of  $\bar{F}_{zm}$ , at  $V_0 = 0.03 \text{ m/s}$ , with  $A_w(\rho_w - \rho_I) gh + \sigma_f h^2$ .



(a) Force ratio versus hull velocity



(b) Moment arm versus hull velocity

Figure 12. Variation of force ratio  $\alpha_1$  and moment arm  $L_I$  with hull velocity,  $V_0$ .

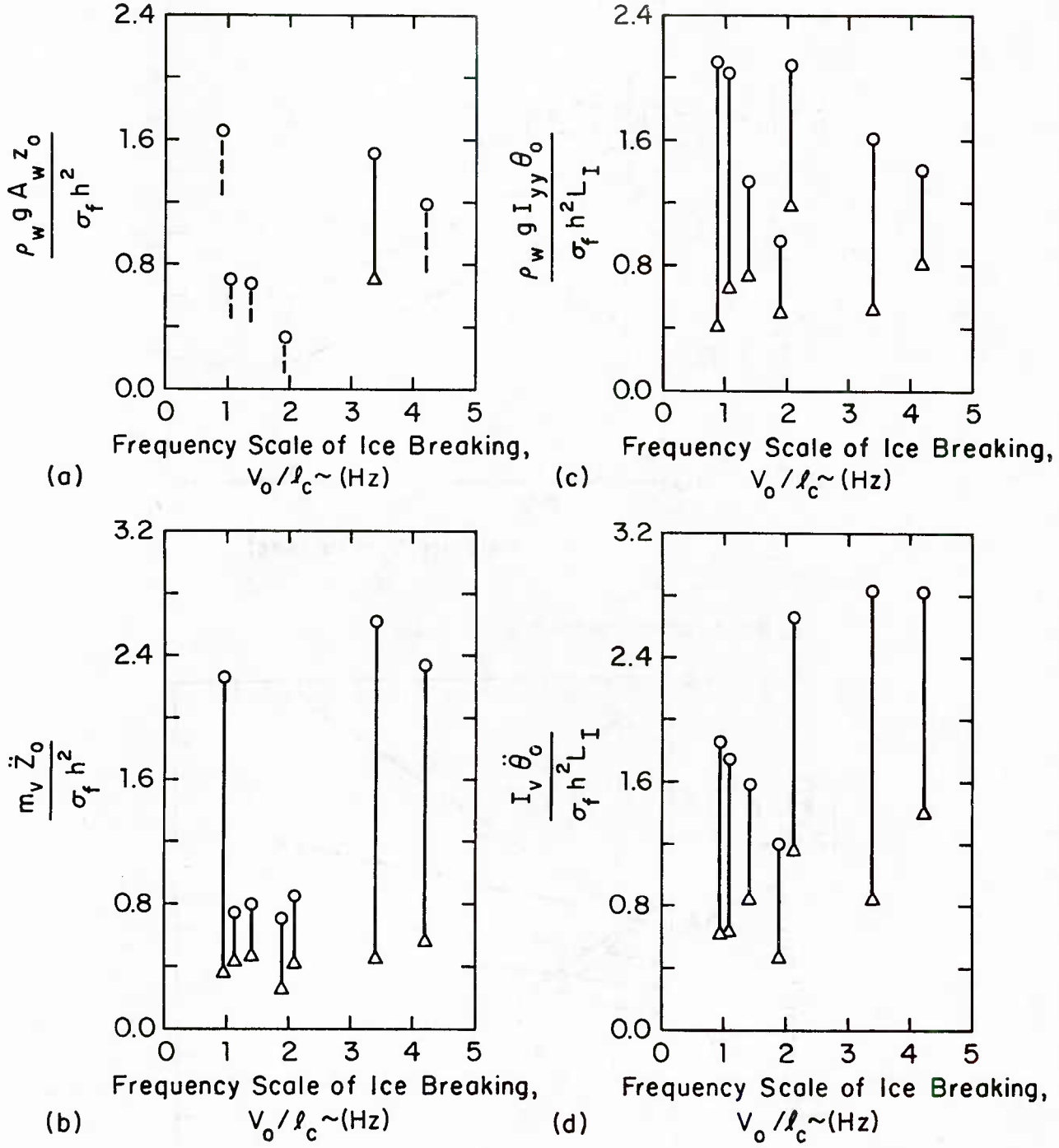


Figure 13. Typical maxima (O) and minima ( $\Delta$ ) of peak-to-peak values of vertical forces and moments plotted versus frequency scale of ice breaking,  $V_o/l_c$ .



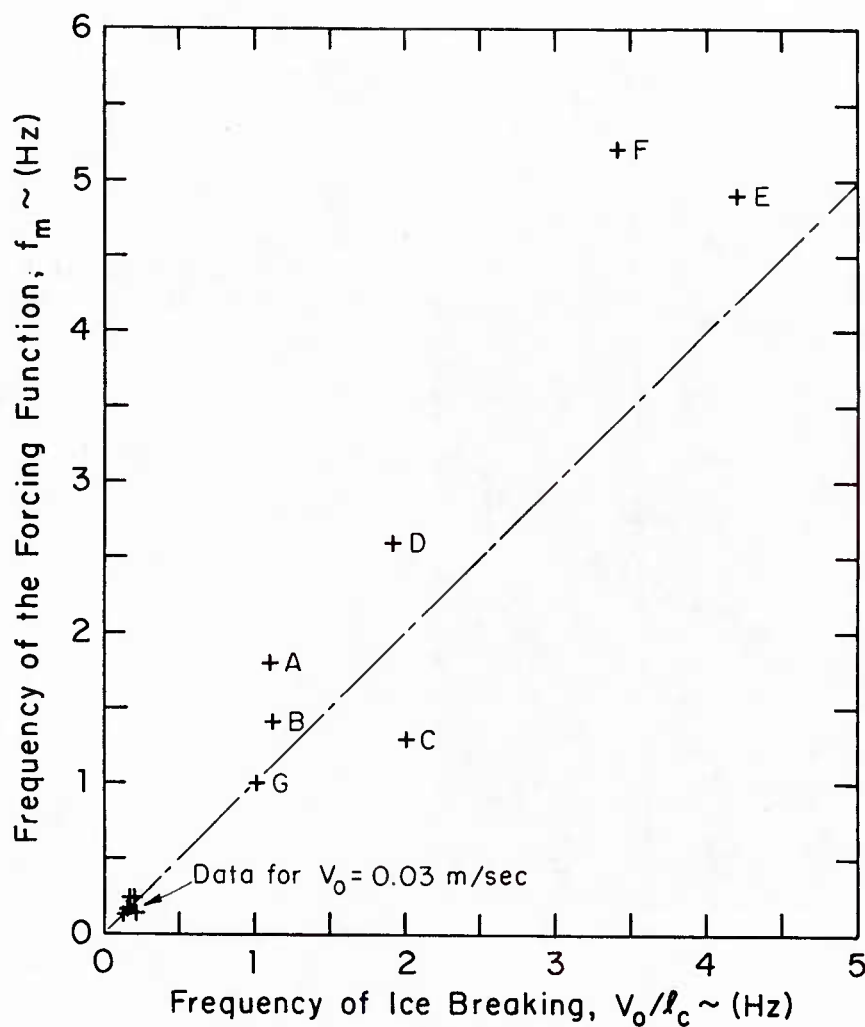


Figure 14. Correlation of frequency of forcing function,  $f_m$ , with frequency-scale of the breaking,  $V_0/l_c$ .

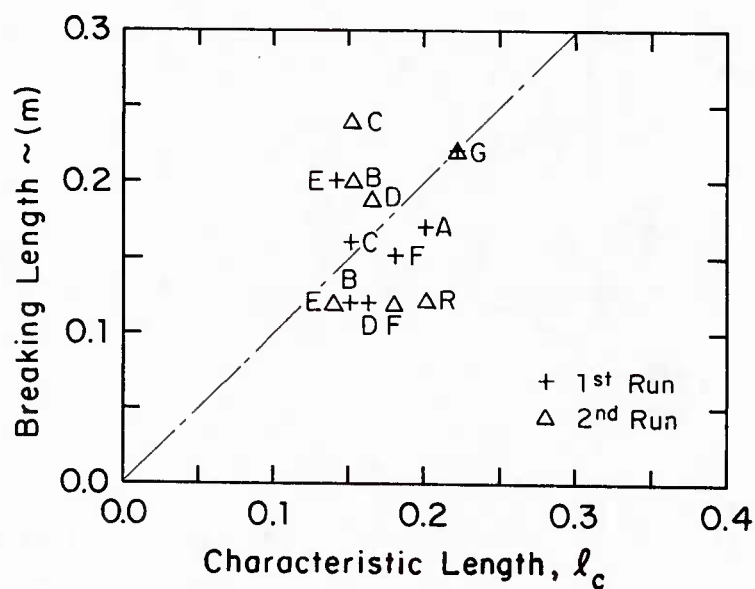
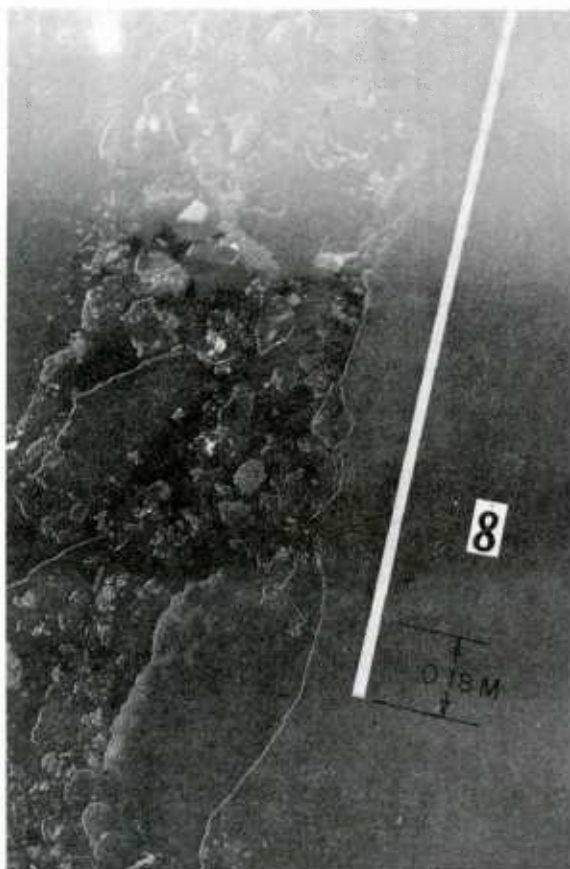
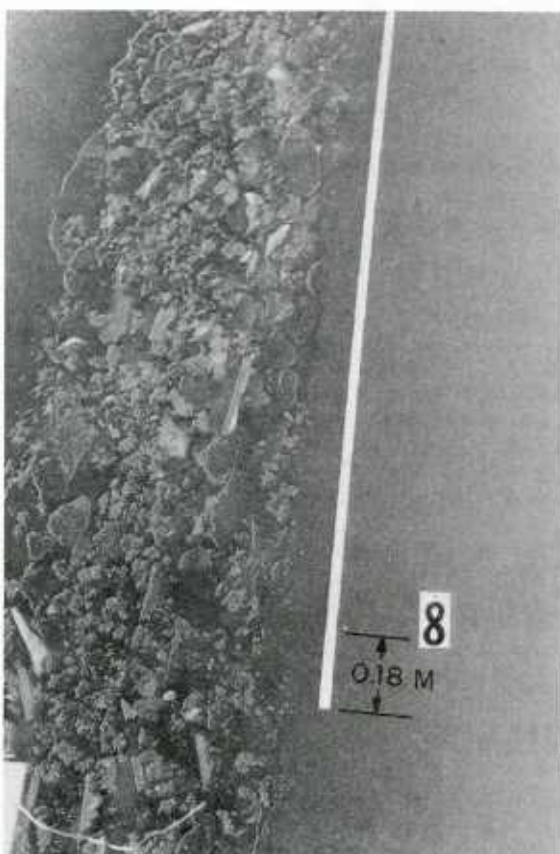


Figure 15. Correlation of ice-breaking length,  $\Delta x$ , with characteristic length of ice sheet,  $l_c$ .



(a) Hull moving at a relatively low velocity  $V_o = 0.03$  m/s



(b) Hull moving at a relatively high velocity  $V_o = 0.60$  m/s

Figure 16. Typical sizes of ice rubble after the model hull has transmitted ice sheets.

Table 1. List of Physical Parameters of Ship Model

Overall length of hull	2.534 m
Length between perpendicular	2.235 m
Beam (maximum width)	0.530 m
Beam (design at waterline)	0.495 m
Draft (design)	0.178 m
Displacement	$m = 102.6 \text{ kg}$
Virtual mass coefficient (heaving motion)	$a = 1.38$
Mass moment of inertia	$I = 33.3 \text{ kgm}^2$
Virtual mass moment of inertia coefficient	$A = 0.55$
Area of the waterplane of the hull	$A_w = 0.82 \text{ m}^2$
Area moment of inertia for pitch motion	$I_{yy} = 0.25 \text{ m}^4$
Wetted area of hull	$S = 1.32 \text{ m}^2$

Table 2. Measurement of Open-Water Resistance to Hull Motion

Hull Velocity	Froude No.	Reynolds No.	Resistance Force	Coefficient of Resistance
$V_O$ (m/sec)	$\frac{V_O}{\sqrt{gL_S}}$	$\frac{V_O L_S}{\nu}$	$F_x$ (N)	$C$
0.03	0.0064	$37 \times 10^3$	0.2	0.34
0.15	0.032	$188 \times 10^3$	0.4	0.023
0.22	0.046	$276 \times 10^3$	0.4	0.012
0.30	0.064	$377 \times 10^3$	0.5	0.0084
0.59	0.128	$741 \times 10^3$	1.5	0.0065

Table 3. List of the Bandwidths and the Sensitivities of the Instruments

	Bandwidth		Sensitivity
	Range (Hz)	Limited by	
Force $F_x$	0-3	digital filtering to remove resonant frequency and 6 Hz	33.5 N/Volt
Accelerometers	0-6	analog-filter before A/D conversion	0.42 m/sec <sup>2</sup> /volt
.. $z(t)$	0.5-6	Rolling of hull	
Linear Potentiometers	0 - $\infty$		0.013 m/Volt
Velocity	0	Speed control of the carriage	0.137 m/sec/Volt

Table 4. List of Experimental Errors

Parameter	Error	Main Source of Error
$F_x$	$\pm 1N$	electrical drift
Draft $z$	$< \pm 0.00065 \text{ m}$	static friction of
$\rho_w g A_w z$	$\pm 5N$	ball bushings
Pitch $\theta$	$\pm 0.0004 \text{ radius}$	electrical drift
$\rho_w g I_{yy} \theta$	$+ 1Nm$	
$\ddot{z}$	$\pm 0.02 \text{ m/sec}^2$	Rolling of Ship
$m_V \ddot{z}$	$\pm 5N$	$\ddot{z} = 0.005 \text{ m/sec}^2/\text{degree}$ error occuring at low frequencies)
$\ddot{\theta}$	$\pm 0.01 \text{ sec}^{-2}$	electrical drift
$I_V \ddot{\theta}$	$\pm 0.5 \text{ Nm}$	

Table 5. Physical Parameters of Ice Sheets and Test Runs

Sheet	Date	$h$ (m)	$\sigma_f$ (kPa)	$E_f$ (MPa)	$l_c$ (m)	$\sigma_f h^2$ (N)	Run No	$V_o$ (m/sec)	$\frac{V_o}{I_c}$ (sec <sup>-1</sup> )	$\frac{V_o}{f_o I_c}$	$T$ (sec)
A	10/15/82	0.25	23	11	0.20	14.4	02015	0.03	0.15	0.15	47.7
B	10/27/82	0.021	29	6.5	0.15	12.9	03015	0.22	1.1	1.1	36.0
							04027	0.03	0.20	0.20	48.9
C	10/18/82	0.023	10	4.2	0.15	5.3	06027	0.22	1.5	1.4	19.2
							02018	0.03	0.20	0.20	42.9
D	10/25/82	0.023	25	6.0	0.16	13.0	03018	0.30	2.0	2.0	29.8
							04025	0.03	0.19	0.19	44.1
E	10/20/82	0.022	19	3.8	0.14	9.3	05025	0.30	1.9	1.9	30.0
							04020	0.03	0.21	0.21	59.1
F	10/29/82	0.023	29	8.5	0.18	15.3	05020	0.59	4.2	4.2	6.0
							06029	0.03	0.17	0.17	49.1
G	10/22/82	0.032	18	8.3	0.22	18.1	07029	0.60	3.4	3.4	12.0
							04022	0.03	0.13	0.13	49.1
							05022	0.22	1.0	1.0	27.0



Table 6. List of Scaling Factors

Draft:	$\rho_w g A_w z = 7950 z \text{ (N)}$
Vertical Acceleration:	$m_V \ddot{z} = 244 \ddot{z} \text{ (N)}$
Pitch:	$\rho_w g I_{YY} \theta = 2450 \theta \text{ (Nm)}$
Angular Acceleration:	$I_V \ddot{\theta} = 51.6 \ddot{\theta} \text{ (Nm)}$

Table 7. Effect of the Depression of the Water Level at the Bow

Sheet	$V_O$ (m/sec)	$\bar{F}_x$ (N)	$\bar{F}_z$ (N)	$\bar{M}_y$ (Nm)	$\alpha_1$	$L_I$ (m)	$\Delta B'_z$ (N)	$\Delta z$ (m)	$\Delta M'_y$ (Nm)	$L$ (m)	$\Delta B'_x$ (N)
E	0.59	18.6	8.74	13.72	0.47	1.56	19.2	0.024	6.7	0.35	1.5
F	0.60	26.5	19.9	20.8	0.75	1.04	19.9	0.025	8.2	0.41	1.5

## APPENDIX A

Mean, Standard Deviations, Covariances and Correlation Coefficients  
of the Experimental Runs

## Nomenclature:

Draft  $z(t)$  $A_z$   $\ddot{z}(t)$ Pitch  $\theta(t)$ Omega  $\ddot{\theta}(t)$

SHEET A VEL = 0.03 (RUN NO : 02015 )

	Fx N	Draft m	Az m/sec**2	Pitch radians	Omega 1/sec**2
MEAN	13.64	0.0026	-0.003	0.0043	-0.007
SIGMA	3.41	0.0002	0.003	0.0009	0.004

scaled as forces and moments

	N	N	N	Nm	Nm
MEAN	13.64	20.59	-0.81	10.46	-0.38
SIGMA	3.41	1.27	0.71	2.31	0.21

Covariance of

Fx with : (N*N) or (N*Nm)	-0.58	-0.83	4.35	-0.09
Draft with : (N*N) or (N*Nm)		-0.20	1.60	-0.07
Az with : (N*Nm)			-0.79	0.02
Pitch with : (Nm*Nm)				-0.17

Corr. Coefficient

Fx with :	-0.13	-0.35	0.55	-0.13
Draft with :		-0.23	0.54	-0.26
Az with :			-0.48	0.15
Pitch with :				-0.35

SHEET A VEL = 0.22 (RUN NO : 03015 )

	Fx N	Draft m	Az m/sec**2	Pitch radians	Omega 1/sec**2
MEAN	13.59	0.0025	-0.012	0.0050	-0.008
SIGMA	3.76	0.0004	0.010	0.0017	0.050

scaled as forces and moments

	N	N	N	Nm	Nm
MEAN	13.59	20.19	-2.92	12.27	-0.40
SIGMA	3.76	3.15	2.48	4.23	2.57

Covariance of

Fx with : (N*N) or (N*Nm)	7.16	-0.86	9.19	-2.77
Draft with : (N*N) or (N*Nm)		0.08	8.48	-1.71
Az with : (N*Nm)			0.43	0.46
Pitch with : (Nm*Nm)				-7.87

Corr. Coefficient

Fx with :	0.60	-0.09	0.58	-0.29
Draft with :		0.01	0.64	-0.21
Az with :			0.04	0.07
Pitch with :				-0.72

SHEET B VEL = 0.03 (RUN NO : 04027 )

	Fx N	Draft m	Az m/sec**2	Pitch radians	Omega 1/sec**2
MEAN	8.06	0.0017	0.004	0.0046	-0.004
SIGMA	1.65	0.0002	0.005	0.0008	0.011

scaled as forces and moments

	N	N	N	Nm	Nm
MEAN	8.06	13.12	0.92	11.20	-0.23
SIGMA	1.65	1.32	1.24	2.03	0.58

Covariance of

Fx with : (N*N) or (N*Nm)	0.52	-0.47	1.32	-0.05
Draft with : (N*N) or (N*Nm)		-1.14	1.80	-0.16
Az with : (N*Nm)			-1.30	0.06
Pitch with : (Nm*Nm)				-0.53

Corr. Coefficient

Fx with :	0.24	-0.23	0.39	-0.05
Draft with :		-0.70	0.67	-0.21
Az with :			-0.52	0.08
Pitch with :				-0.45

SHEET B VEL = 0.22 (RUN NO : 06027 )

	Fx N	Draft m	Az m/sec**2	Pitch radians	Omega 1/sec**2
MEAN	15.12	0.0027	-0.001	0.0047	-0.001
SIGMA	3.60	0.0003	0.008	0.0014	0.059

scaled as forces and moments

	N	N	N	Nm	Nm
MEAN	15.12	21.51	-0.26	11.64	-0.04
SIGMA	3.60	2.61	1.91	3.33	3.05

Covariance of

Fx with : (N*N) or (N*Nm)	0.62	-2.43	5.17	-2.77
Draft with : (N*N) or (N*Nm)		-0.91	2.67	-1.66
Az with : (N*Nm)			-2.73	3.00
Pitch with : (Nm*Nm)				-7.39

Corr. Coefficient

Fx with :	0.07	-0.35	0.43	-0.25
Draft with :		-0.18	0.31	-0.21
Az with :			-0.43	0.51
Pitch with :				-0.73

SHEET C VEL = 0.03 (RUN NO : 02018 )

	Fx N	Draft m	Az m/sec**2	Pitch radians	Omega 1/sec**2
MEAN	7.73	0.0014	0.004	0.0038	-0.002
SIGMA	2.01	0.0001	0.003	0.0006	0.004

scaled as forces and moments

	N	N	N	Nm	Nm
MEAN	7.73	10.94	1.06	9.21	-0.12
SIGMA	2.01	0.66	0.68	1.35	0.23

Covariance of

Fx with : (N*N) or (N*Nm)	-0.22	-0.10	1.77	-0.03
Draft with : (N*N) or (N*Nm)		0.07	0.12	-0.02
Az with : (N*Nm)			-0.06	-0.03
Pitch with : (Nm*Nm)				-0.10

Corr. Coefficient

Fx with :	-0.16	-0.08	0.65	-0.06
Draft with :		0.16	0.13	-0.10
Az with :			-0.06	-0.20
Pitch with :				-0.31

SHEET C VEL = 0.30 (RUN NO : 03018 )

	Fx N	Draft m	Az m/sec**2	Pitch radians	Omega 1/sec**2
MEAN	11.73	0.0020	0.003	0.0032	-0.002
SIGMA	2.76	0.0003	0.005	0.0009	0.044

scaled as forces and moments

	N	N	N	Nm	Nm
MEAN	11.73	16.01	0.73	7.93	-0.09
SIGMA	2.76	2.45	1.31	2.12	2.27

Covariance of

Fx with : (N*N) or (N*Nm)	1.19	-1.30	2.96	-1.24
Draft with : (N*N) or (N*Nm)		-0.28	1.20	-0.79
Az with : (N*Nm)			-0.83	0.85
Pitch with : (Nm*Nm)				-3.57

Corr. Coefficient

Fx with :	0.18	-0.36	0.51	-0.20
Draft with :		-0.09	0.23	-0.14
Az with :			-0.30	0.29
Pitch with :				-0.74

SHEET D VEL = 0.03 (RUN NO : 04025 )

	Fx N	Draft m	Az m/sec**2	Pitch radians	Omega 1/sec**2
MEAN	11.25	0.0027	-0.004	0.0052	0.003
SIGMA	3.70	0.0002	0.002	0.0015	0.014

scaled as forces and moments

	N	N	N	Nm	Nm
MEAN	11.25	21.17	-1.00	12.65	0.16
SIGMA	3.70	1.24	0.47	3.62	0.74

Covariance of

Fx with : (N*N) or (N*Nm)	1.08	-0.80	7.70	-0.21
Draft with : (N*N) or (N*Nm)		-0.20	2.68	-0.07
Az with : (N*Nm)			-0.95	0.04
Pitch with : (Nm*Nm)				-0.62

Corr. Coefficient

Fx with :	0.23	-0.46	0.58	-0.08
Draft with :		-0.34	0.60	-0.08
Az with :			-0.56	0.11
Pitch with :				-0.23

SHEET D VEL = 0.30 (RUN NO : 05025 )

	Fx N	Draft m	Az m/sec**2	Pitch radians	Omega 1/sec**2
MEAN	15.16	0.0024	0.002	0.0052	-0.003
SIGMA	3.41	0.0004	0.009	0.0013	0.060

scaled as forces and moments

	N	N	N	Nm	Nm
MEAN	15.16	19.11	0.48	12.80	-0.13
SIGMA	3.41	3.23	2.21	3.29	3.10

Covariance of

Fx with : (N*N) or (N*Nm)	4.14	-1.77	5.59	-2.01
Draft with : (N*N) or (N*Nm)		-2.27	7.24	-2.52
Az with : (N*Nm)			-2.72	3.82
Pitch with : (Nm*Nm)				-6.68

Corr. Coefficient

Fx with :	0.38	-0.23	0.50	-0.19
Draft with :		-0.32	0.68	-0.25
Az with :			-0.37	0.56
Pitch with :				-0.66



SHEET E VEL = 0.03 (RUN NO : 04020 )

	Fx N	Draft m	Az m/sec**2	Pitch radians	Omega 1/sec**2
MEAN	10.46	0.0015	0.000	0.0039	0.000
SIGMA	3.34	0.0001	0.001	0.0005	0.003

scaled as forces and moments

	N	N	N	Nm	Nm
MEAN	10.46	12.09	0.01	9.60	0.01
SIGMA	3.34	0.68	0.36	1.26	0.18

Covariance of

Fx with	: (N*N) or (N*Nm)	-0.42	-0.39	1.65	0.00
Draft with	: (N*N) or (N*Nm)		0.02	0.40	-0.01
Az with	: (N*Nm)			0.02	0.01
Pitch with	: (Nm*Nm)				-0.03

Corr. Coefficient

Fx with	:	-0.18	-0.32	0.39	0.00
Draft with	:		0.07	0.47	-0.08
Az with	:			0.05	0.11
Pitch with	:				-0.15

SHEET E VEL = 0.59 (RUN NO : 05020 )

	Fx N	Draft m	Az m/sec**2	Pitch radians	Omega 1/sec**2
MEAN	18.63	0.0011	-0.001	0.0056	0.001
SIGMA	4.10	0.0002	0.013	0.0006	0.056

scaled as forces and moments

	N	N	N	Nm	Nm
MEAN	18.63	8.96	-0.17	13.78	0.03
SIGMA	4.10	1.50	3.18	1.41	2.88

Covariance of

Fx with	: (N*N) or (N*Nm)	0.78	0.36	1.65	0.36
Draft with	: (N*N) or (N*Nm)		-2.16	0.37	-1.24
Az with	: (N*Nm)			-0.79	5.71
Pitch with	: (Nm*Nm)				-2.30

Corr. Coefficient

Fx with	:	0.13	0.03	0.28	0.03
Draft with	:		-0.45	0.18	-0.29
Az with	:			-0.18	0.62
Pitch with	:				-0.57

SHEET F VEL = 0.03 (RUN NO : 06029 )

	Fx N	Draft m	Az m/sec**2	Pitch radians	Omega 1/sec**2
MEAN	13.60	0.0024	-0.003	0.0058	-0.002
SIGMA	3.84	0.0001	0.010	0.0012	0.009

scaled as forces and moments

	N	N	N	Nm	Nm
MEAN	13.60	18.91	-0.75	14.33	-0.11
SIGMA	3.84	0.99	2.56	2.98	0.46

Covariance of

Fx with : (N*N) or (N*Nm)	0.26	-5.84	1.77	0.06
Draft with : (N*N) or (N*Nm)		0.29	0.58	-0.02
Az with : (N*Nm)			-2.93	0.01
Pitch with : (Nm*Nm)				-0.26

Corr. Coefficient

Fx with :	0.07	-0.59	0.16	0.04
Draft with :		0.11	0.20	-0.04
Az with :			-0.39	0.01
Pitch with :				-0.19

SHEET F VEL = 0.60 (RUN NO : 07029 )

	Fx N	Draft m	Az m/sec**2	Pitch radians	Omega 1/sec**2
MEAN	26.54	0.0025	-0.010	0.0085	-0.002
SIGMA	7.25	0.0006	0.031	0.0015	0.085

scaled as forces and moments

	N	N	N	Nm	Nm
MEAN	26.54	19.56	-2.33	20.73	-0.09
SIGMA	7.25	4.72	7.66	3.78	4.39

Covariance of

Fx with : (N*N) or (N*Nm)	12.27	-20.78	10.39	-4.30
Draft with : (N*N) or (N*Nm)		-17.37	14.47	-8.48
Az with : (N*Nm)			-16.05	25.03
Pitch with : (Nm*Nm)				-9.84

Corr. Coefficient

Fx with :	0.36	-0.37	0.38	-0.13
Draft with :		-0.48	0.81	-0.41
Az with :			-0.55	0.74
Pitch with :				-0.59

SHEET G VEL = 0.03 (RUN NO : 04022 )

	Fx N	Draft m	Az m/sec**2	Pitch radians	Omega 1/sec**2
MEAN	19.74	0.0037	0.002	0.0081	-0.004
SIGMA	5.50	0.0005	0.002	0.0018	0.009

scaled as forces and moments

	N	N	N	Nm	Nm
MEAN	19.74	29.08	0.53	19.89	-0.23
SIGMA	5.50	4.10	0.53	4.39	0.45

Covariance of

Fx with	: (N*N) or (N*Nm)	13.31	-1.66	14.12	-0.17
Draft with	: (N*N) or (N*Nm)		-0.70	4.65	-0.11
Az with	: (N*Nm)			-1.57	0.04
Pitch with	: (Nm*Nm)				-0.34

Corr. Coefficient

Fx with	:	0.59	-0.57	0.59	-0.07
Draft with	:		-0.32	0.26	-0.06
Az with	:			-0.67	0.15
Pitch with	:				-0.17

SHEET G VEL = 0.22 (RUN NO : 05022 )

	Fx N	Draft m	Az m/sec**2	Pitch radians	Omega 1/sec**2
MEAN	36.65	0.0048	-0.042	0.0118	-0.001
SIGMA	11.43	0.0008	0.044	0.0027	0.097

scaled as forces and moments

	N	N	N	Nm	Nm
MEAN	36.65	37.96	-10.24	28.83	-0.08
SIGMA	11.43	6.13	10.80	6.71	5.03

Covariance of

Fx with : (N*N) or (N*Nm)	56.55	-91.17	59.71	-16.61
Draft with : (N*N) or (N*Nm)		-48.24	35.16	-14.07
Az with : (N*Nm)			-53.72	22.75
Pitch with : (Nm*Nm)				-22.72

Corr. Coefficient

Fx with :	0.81	-0.74	0.78	-0.29
Draft with :		-0.73	0.85	-0.46
Az with :			-0.74	0.42
Pitch with :				-0.67

## APPENDIX B

## Time Series of the Measured Quantities

Top Figure

 $F_x(t)$  \_\_\_\_\_  
 $\rho_w g A_w z(t)$  - - - - -  
 $m_v z(t)$  - - - - -

Middle Figure

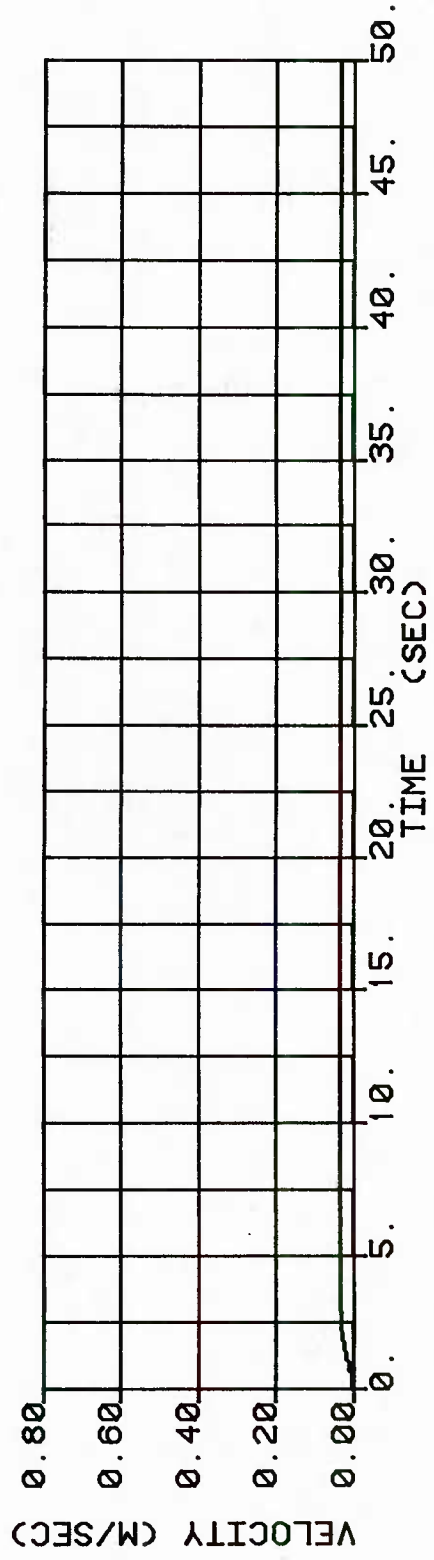
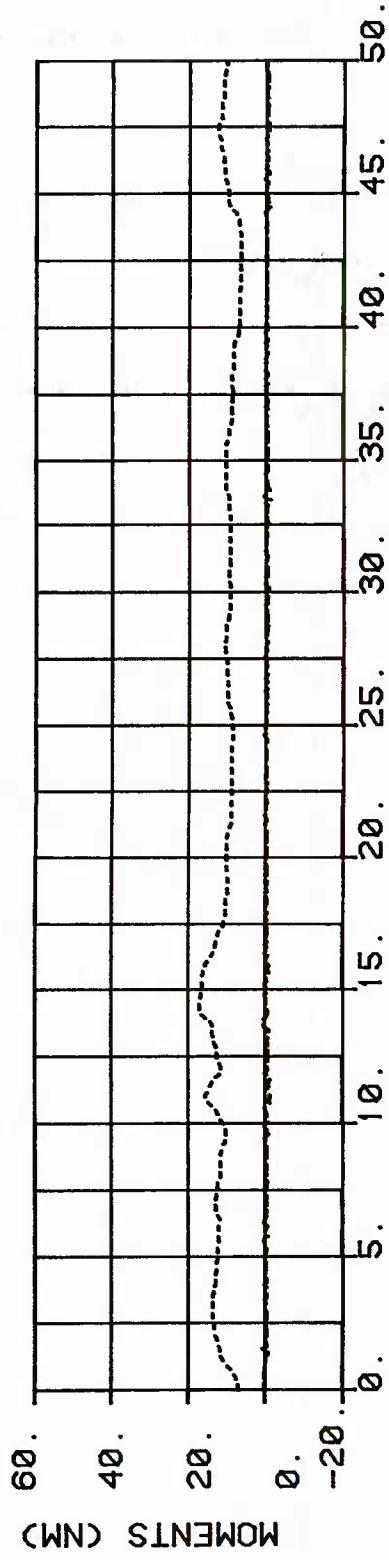
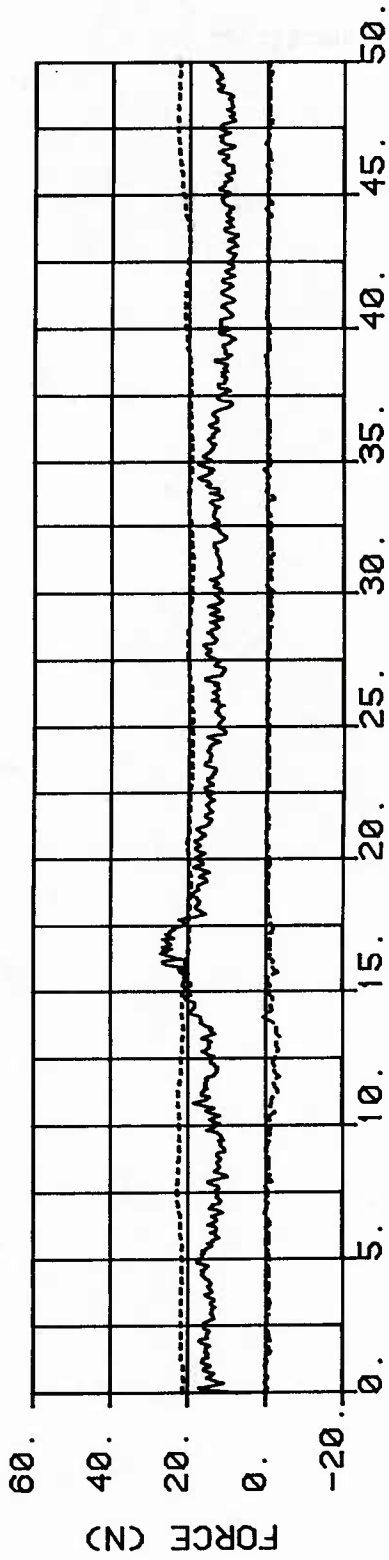
 $\rho_w g I_{yy} \theta(t)$  - - - - -  
 $I_v \theta$  - - - - -

Bottom Figure

 $V_o(t)$  \_\_\_\_\_

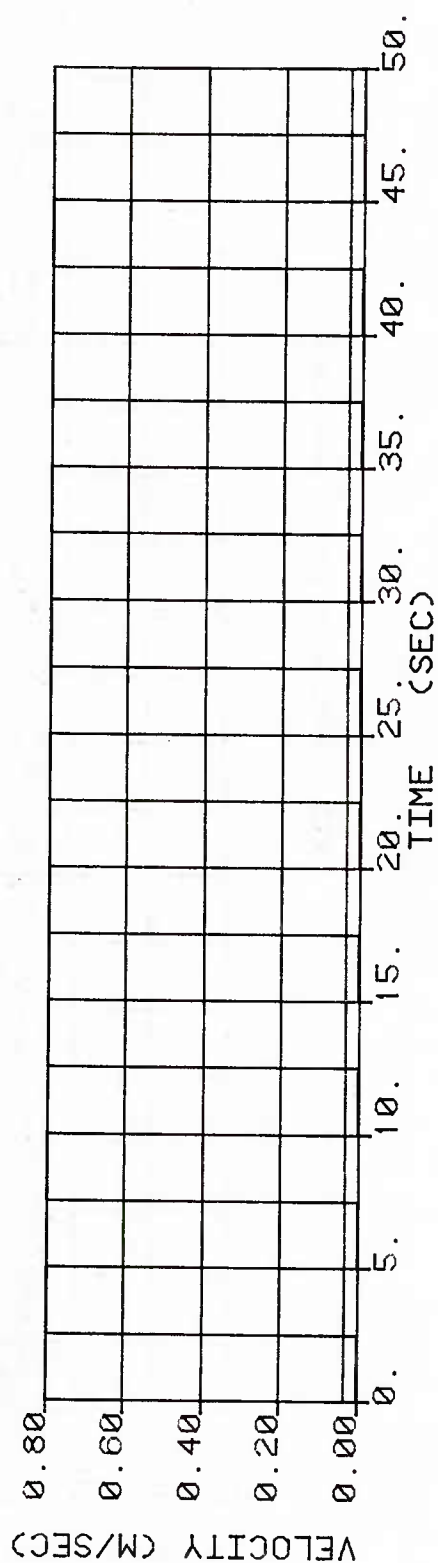
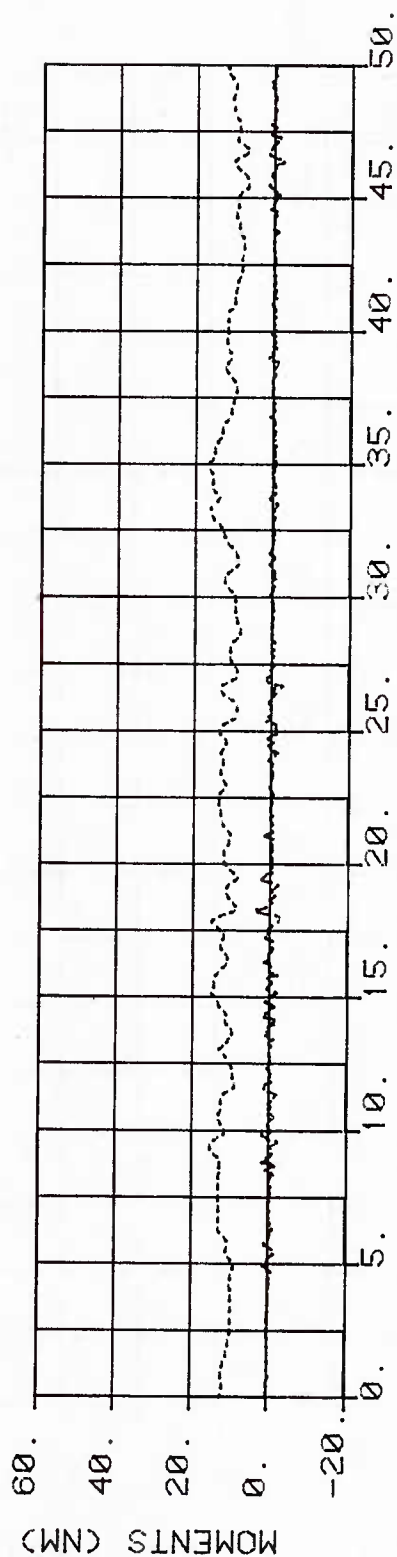
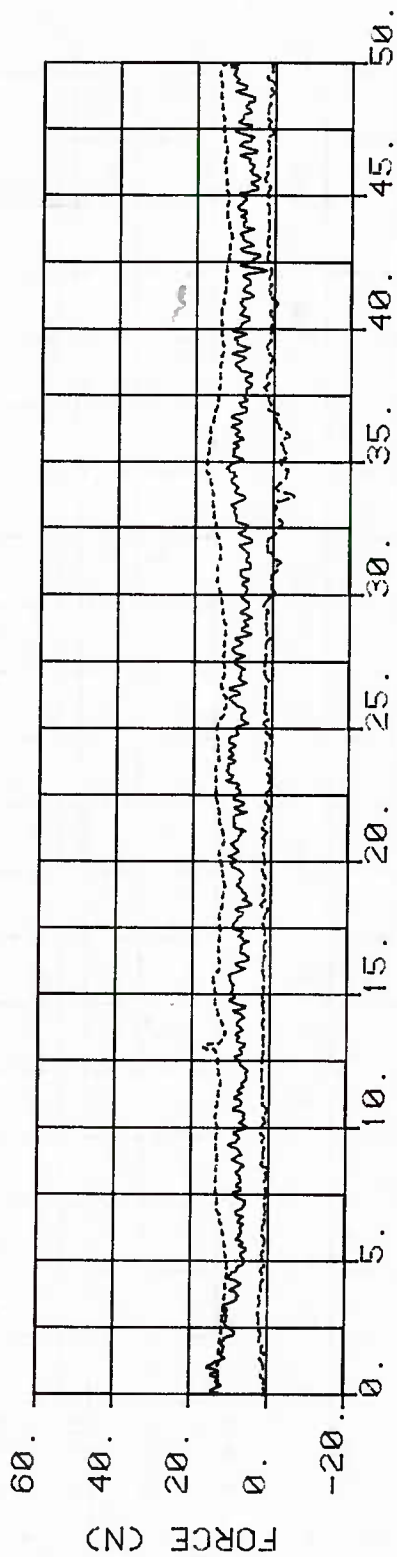
# TIME SERIES

SHEET A : VEL=0.03 M/SEC, RUN 02015



# TIME SERIES

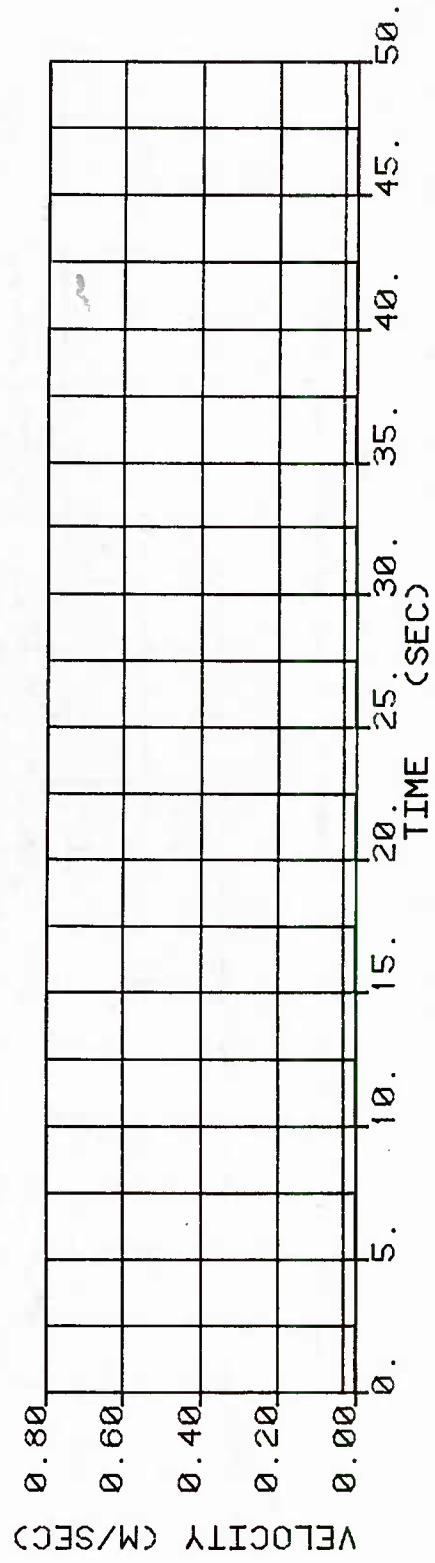
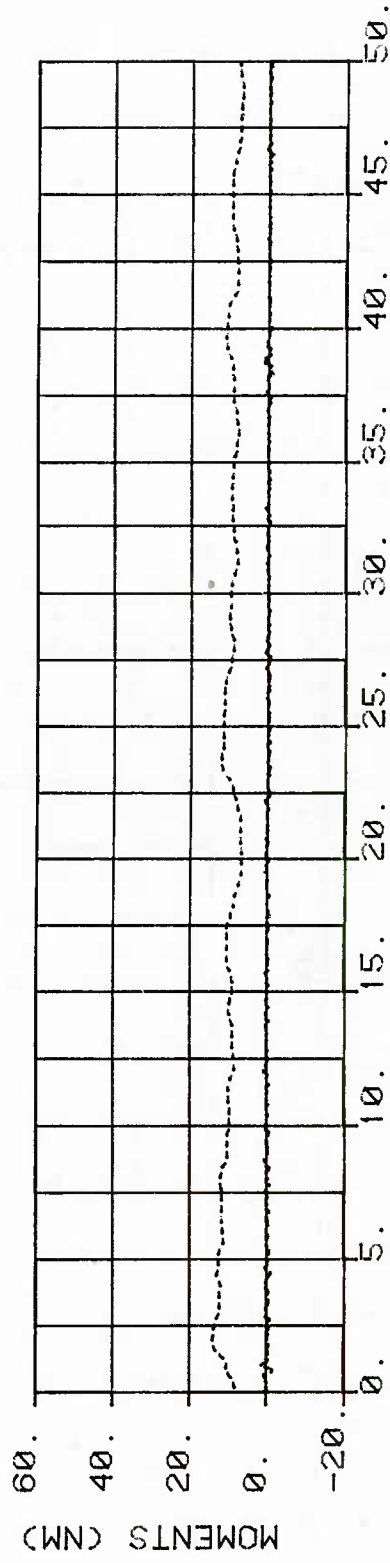
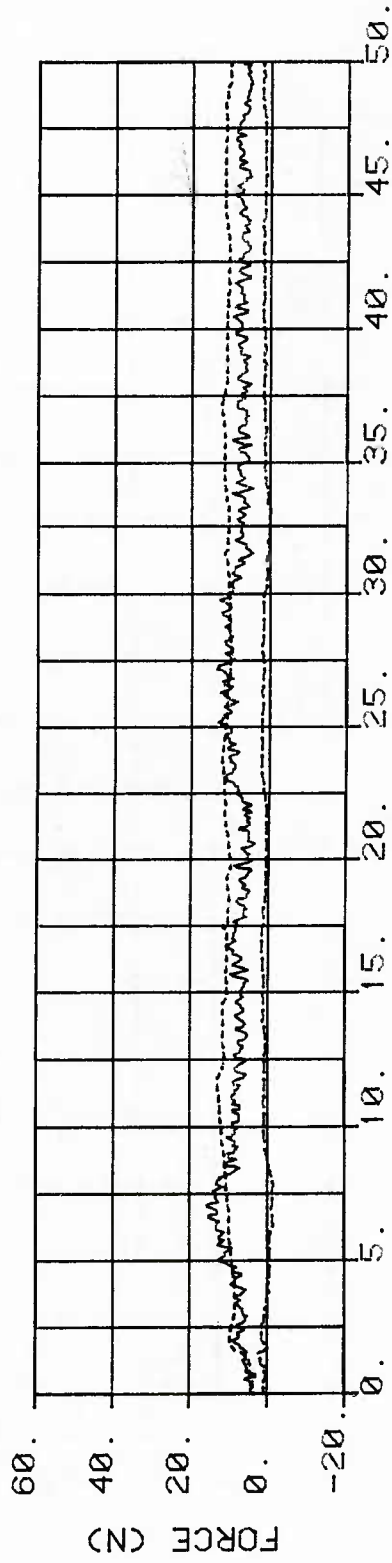
SHEET B : VEL=0.03 M/SEC, RUN 04027





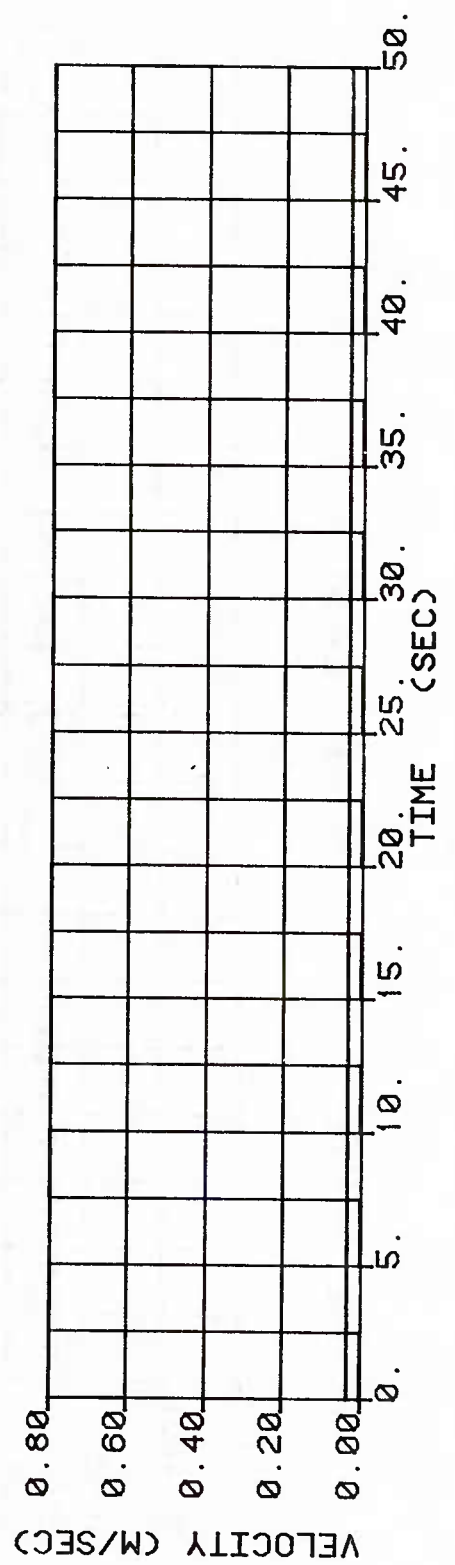
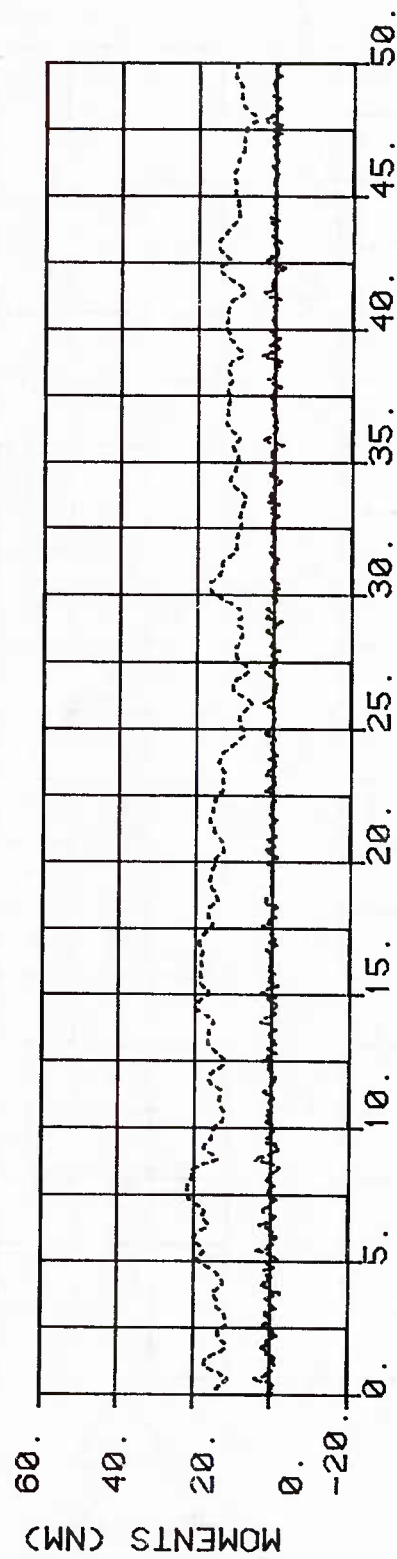
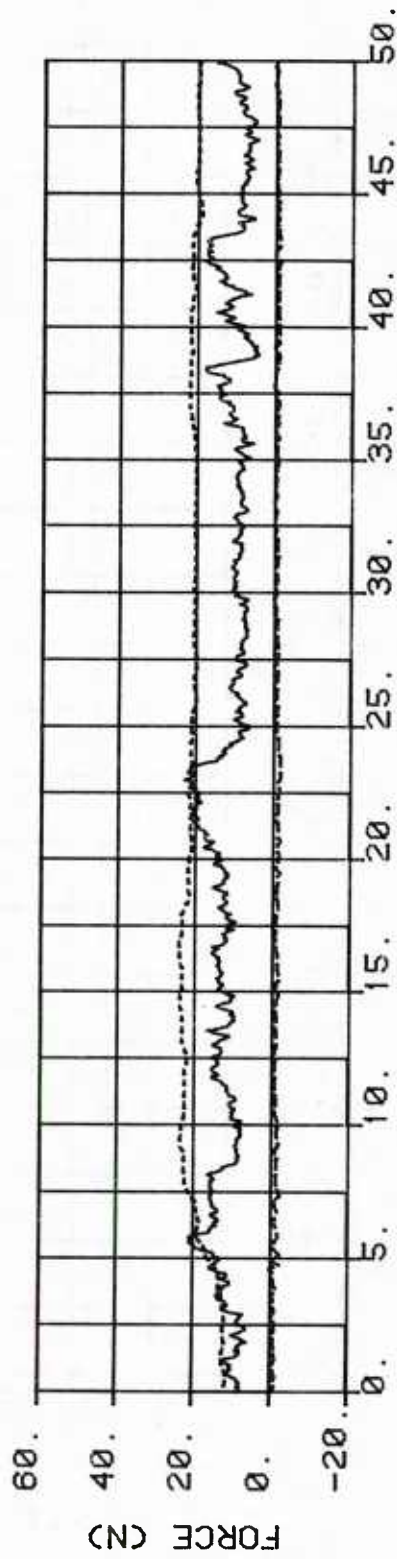
# TIME SERIES

SHEET C : VEL=0.03 M/SEC, RUN 02018



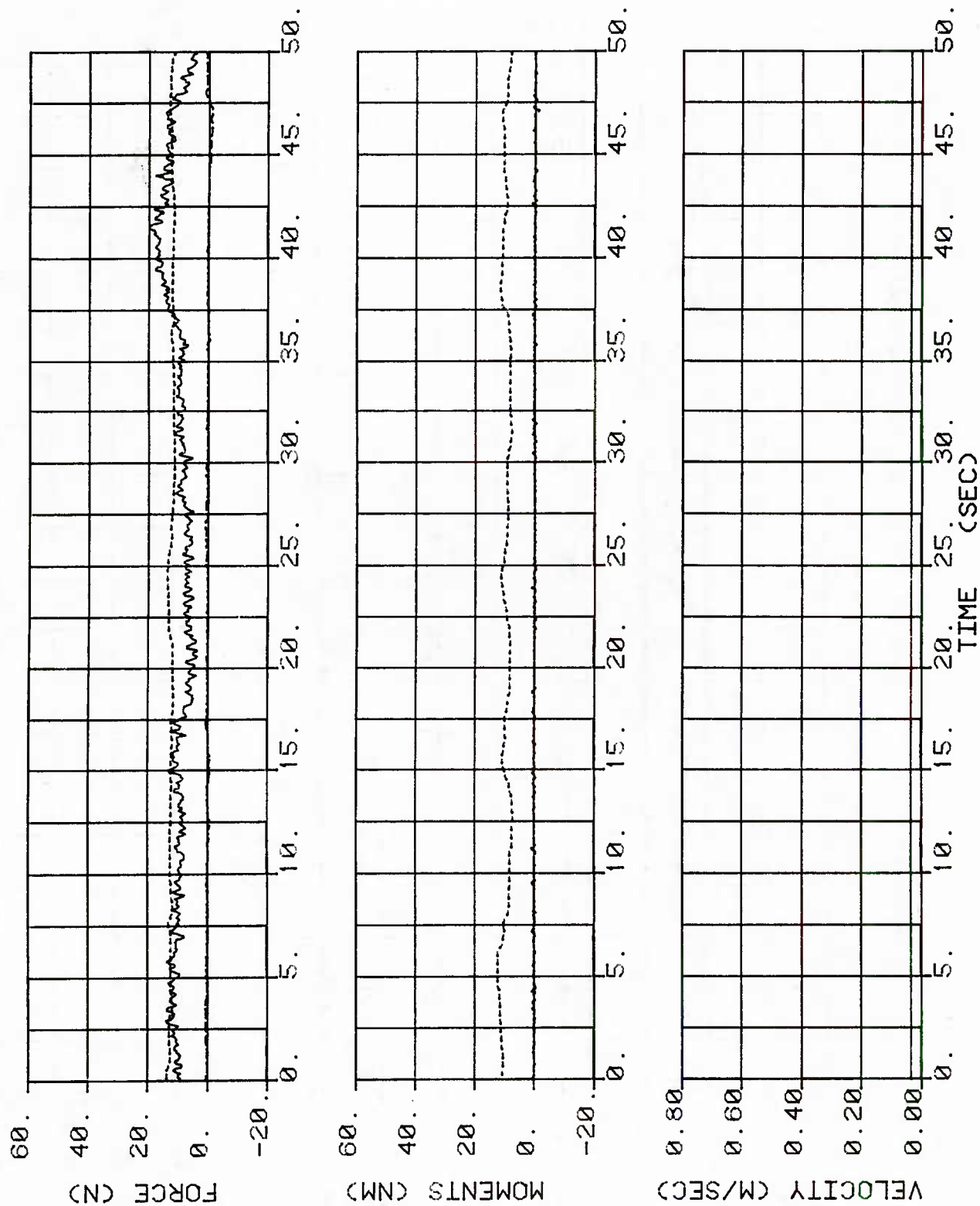
# TIME SERIES

SHEET D : VEL=0.03 M/SEC, RUN 04025



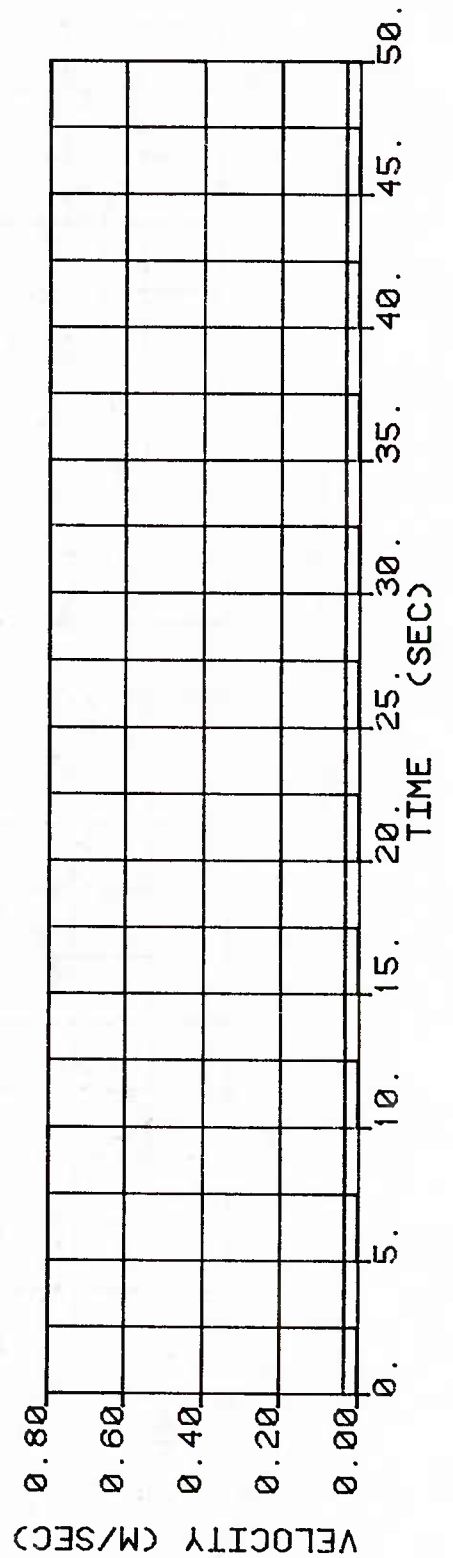
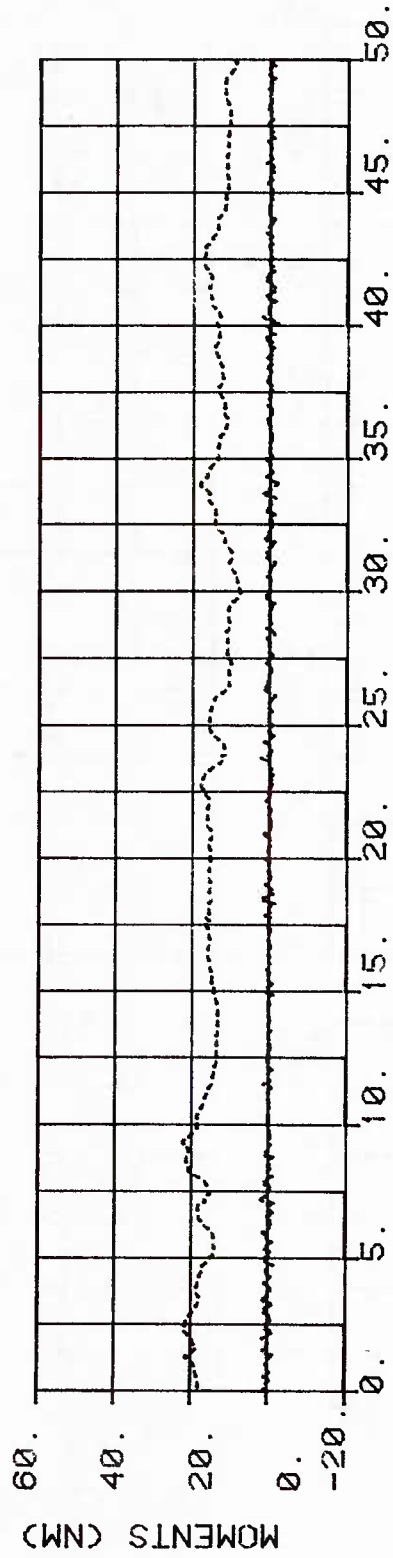
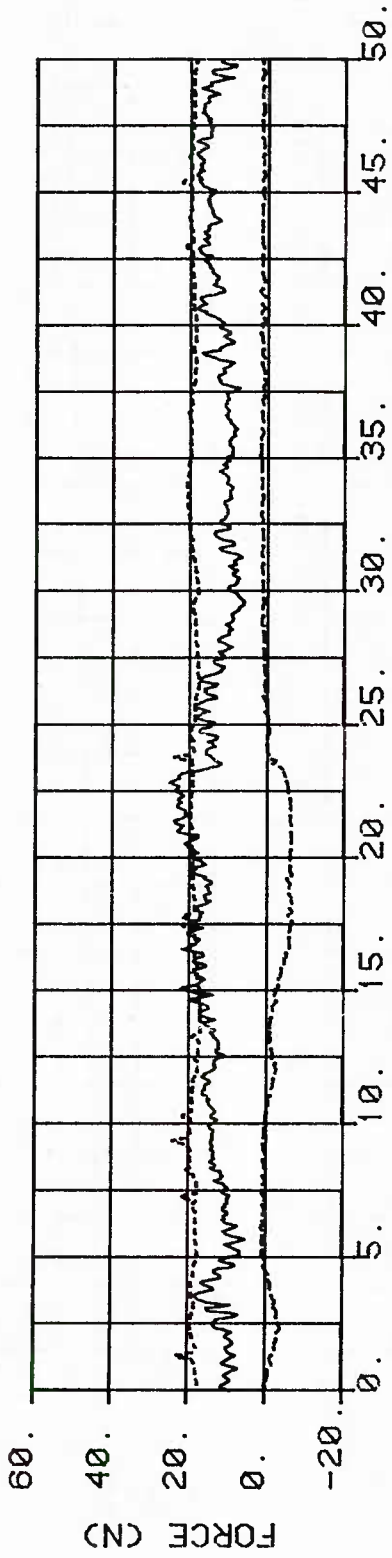
# TIME SERIES

SHEET E : VEL=0.03 M/SEC, RUN 04020



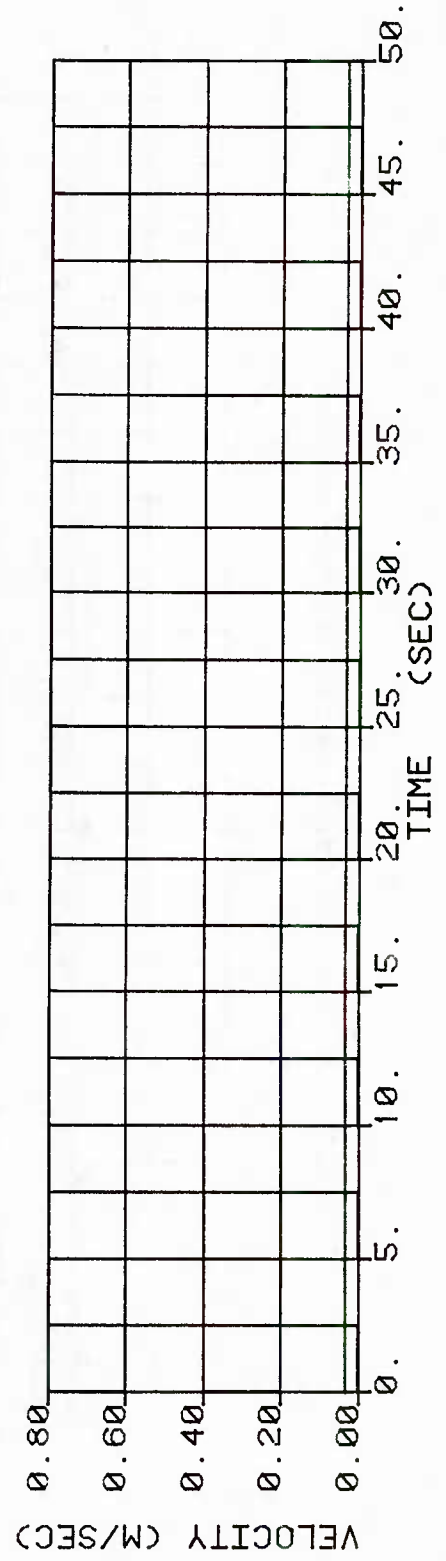
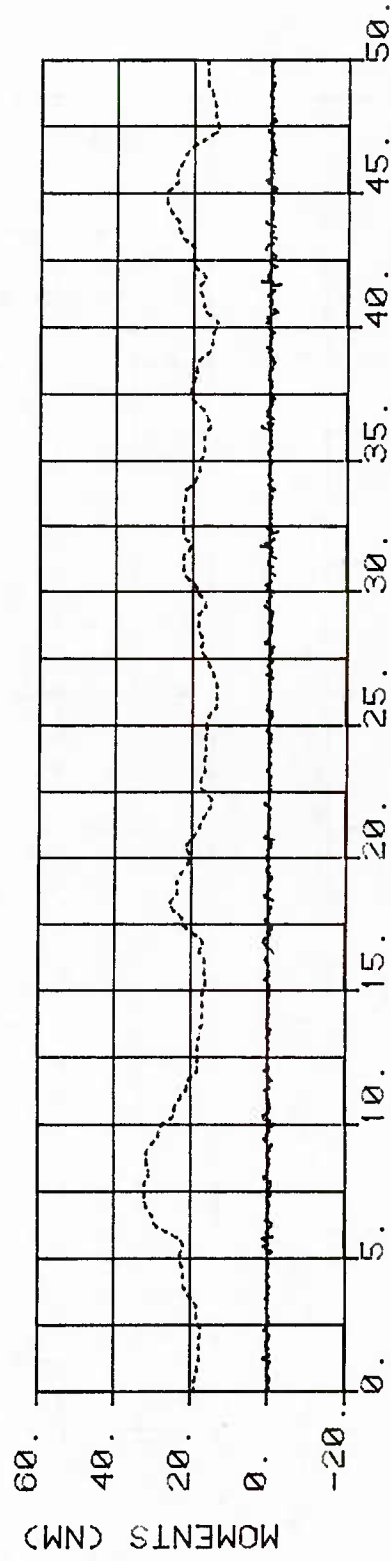
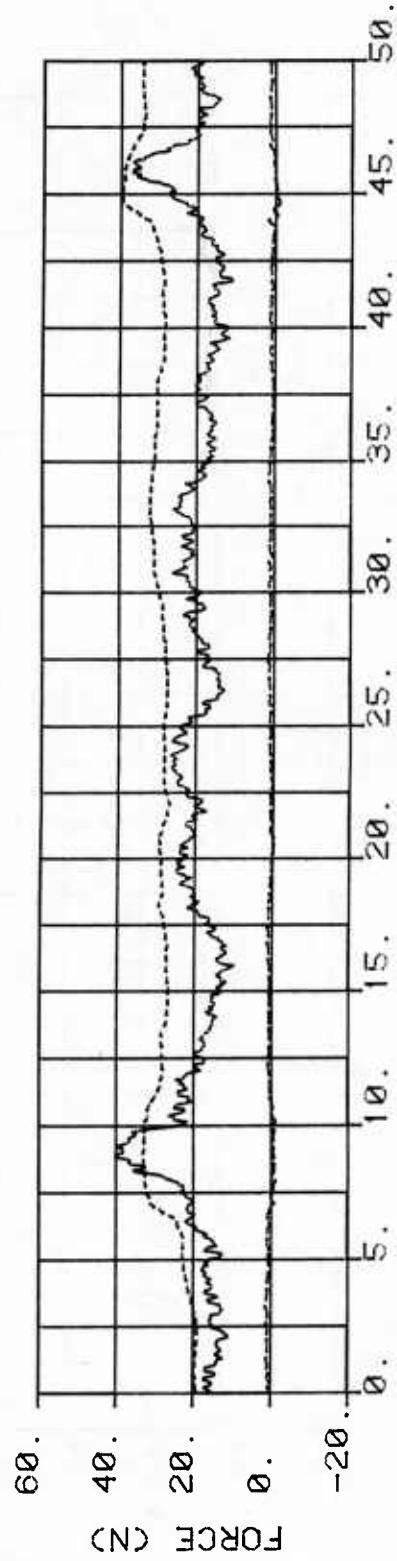
# TIME SERIES

SHEET F : VEL=0.03 M/SEC, RUN 06029



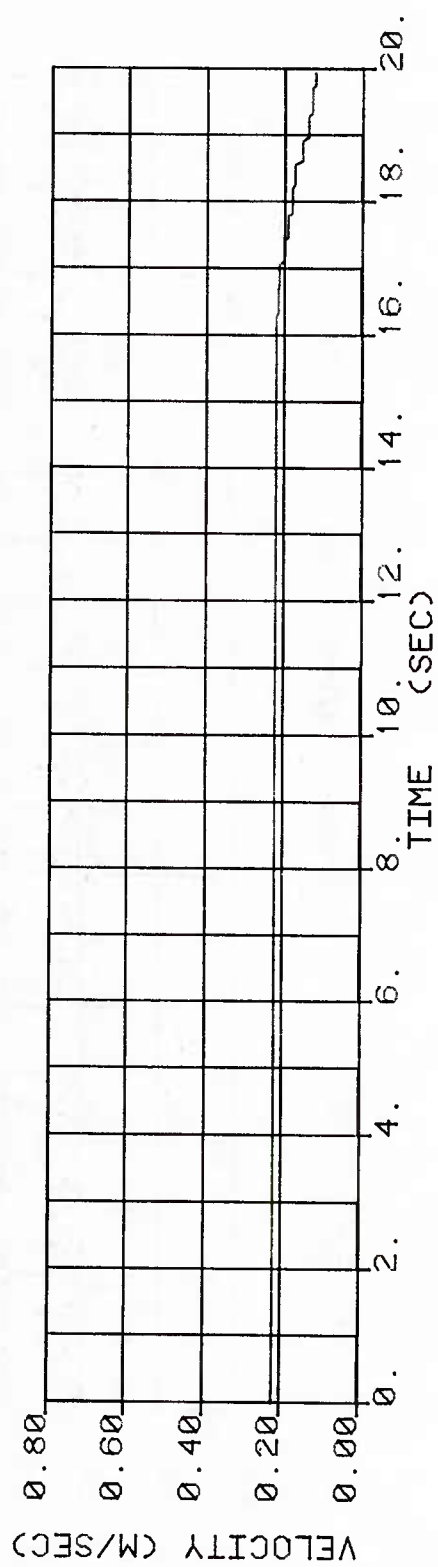
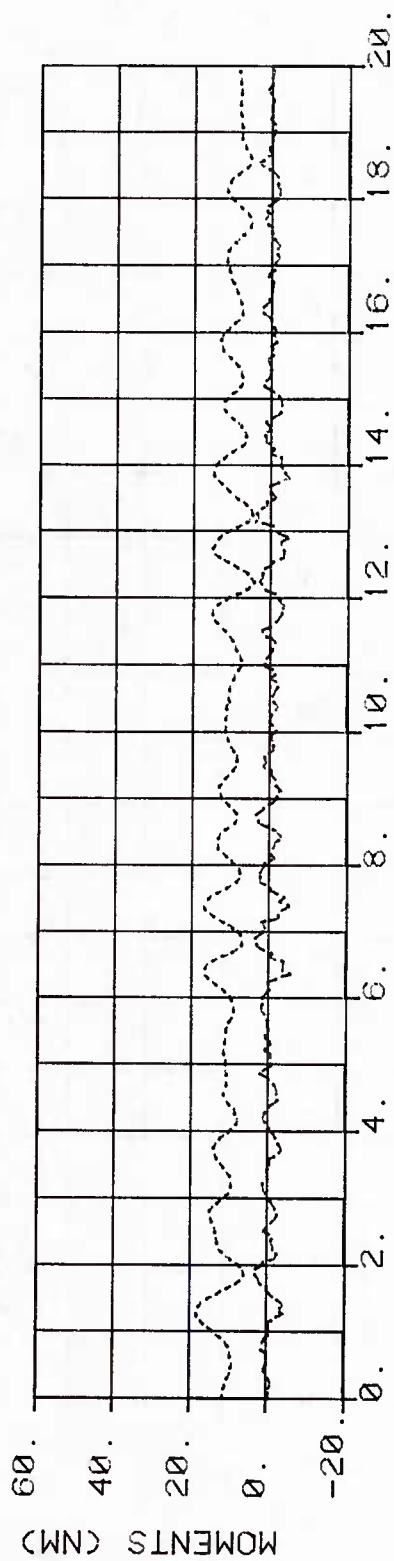
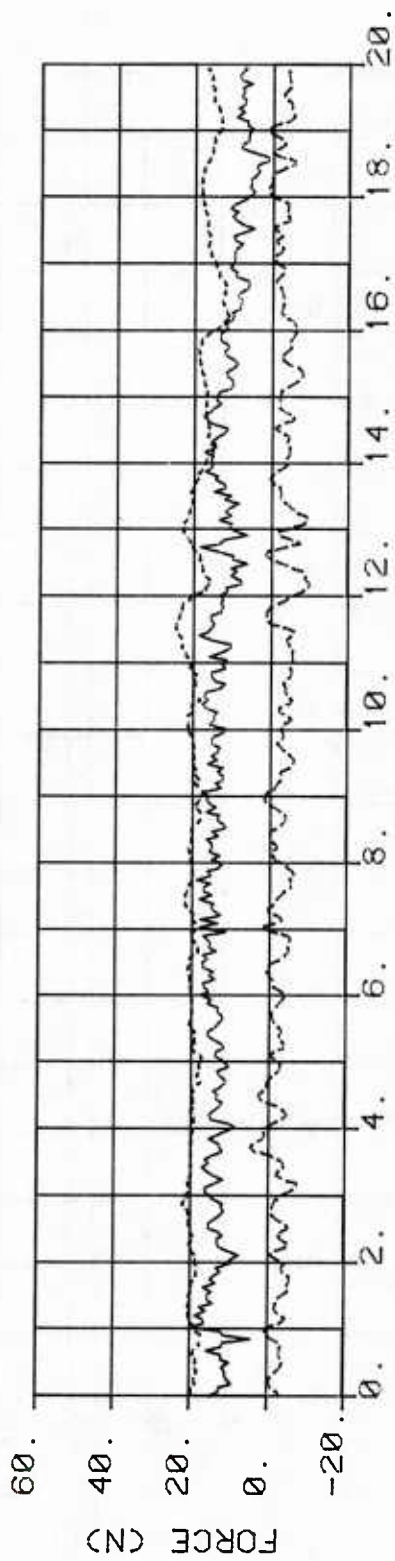
# TIME SERIES

SHEET 6 : VEL=0.03 M/SEC, RUN 04022



# TIME SERIES

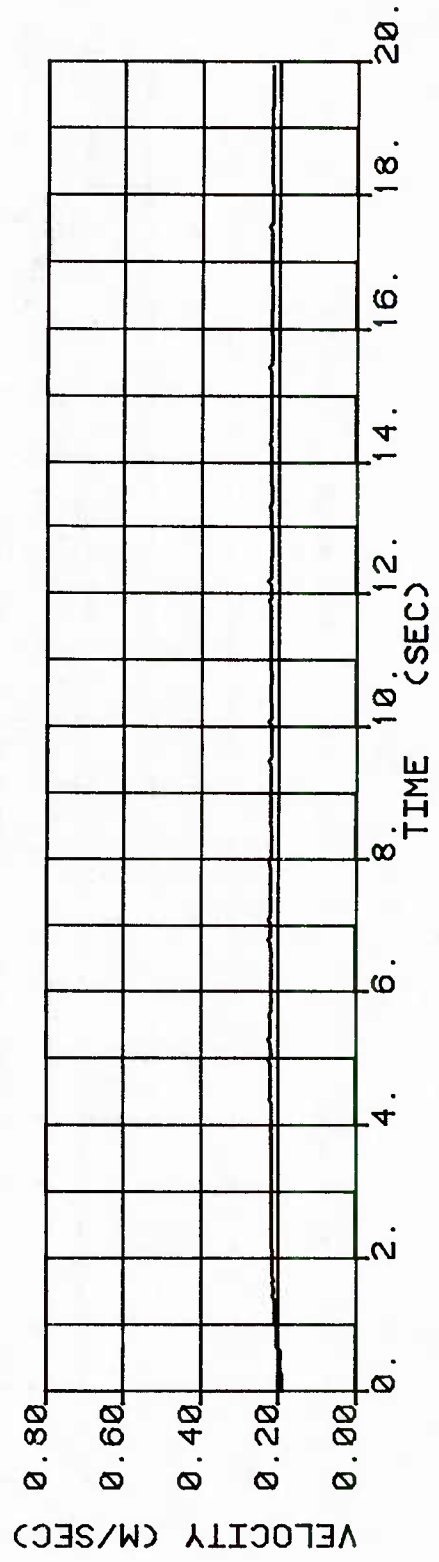
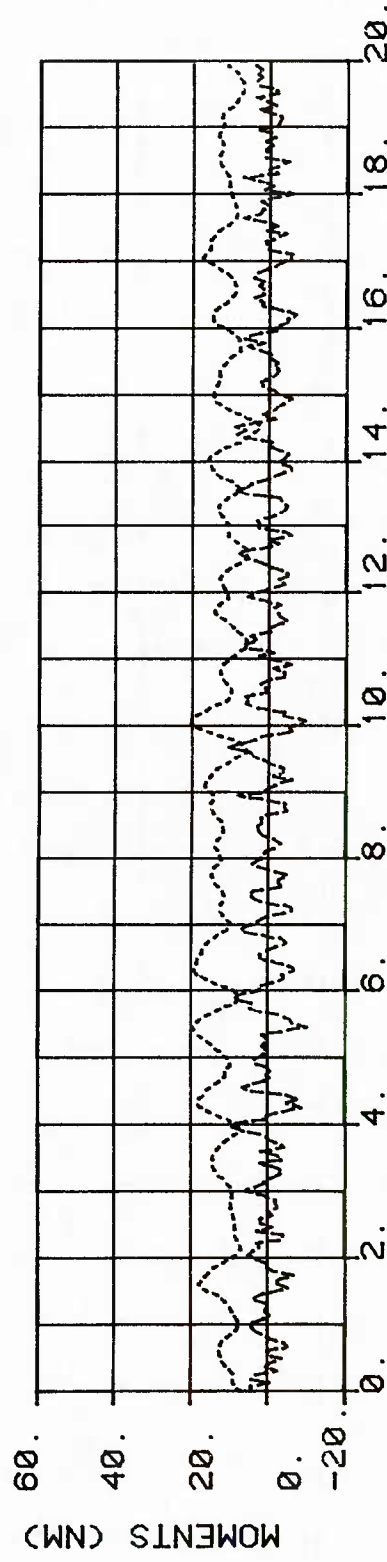
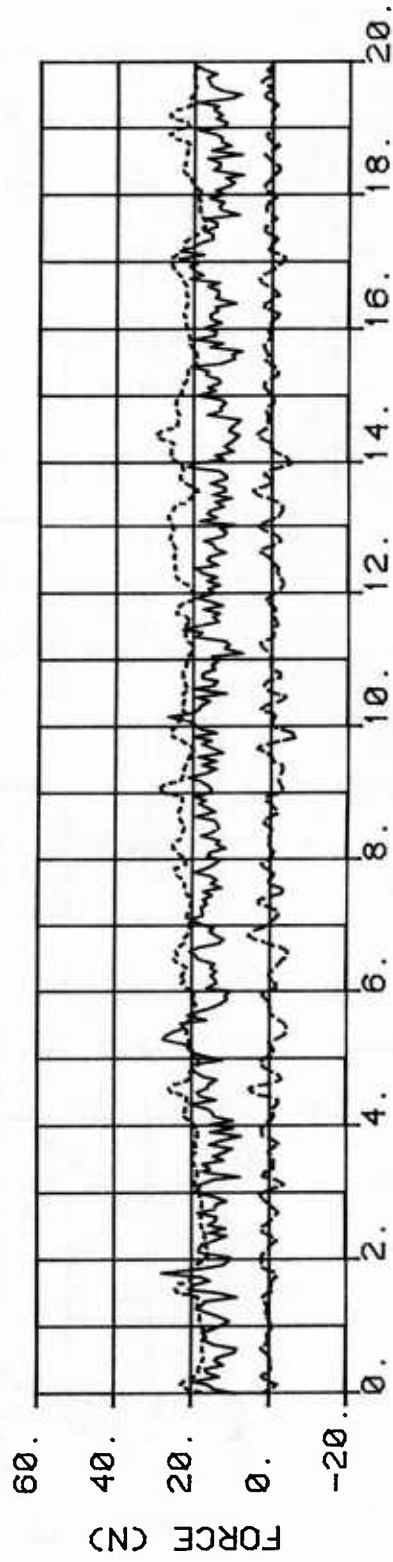
SHEET A : VEL=0.22 M/SEC, RUN 03015





# TIME SERIES

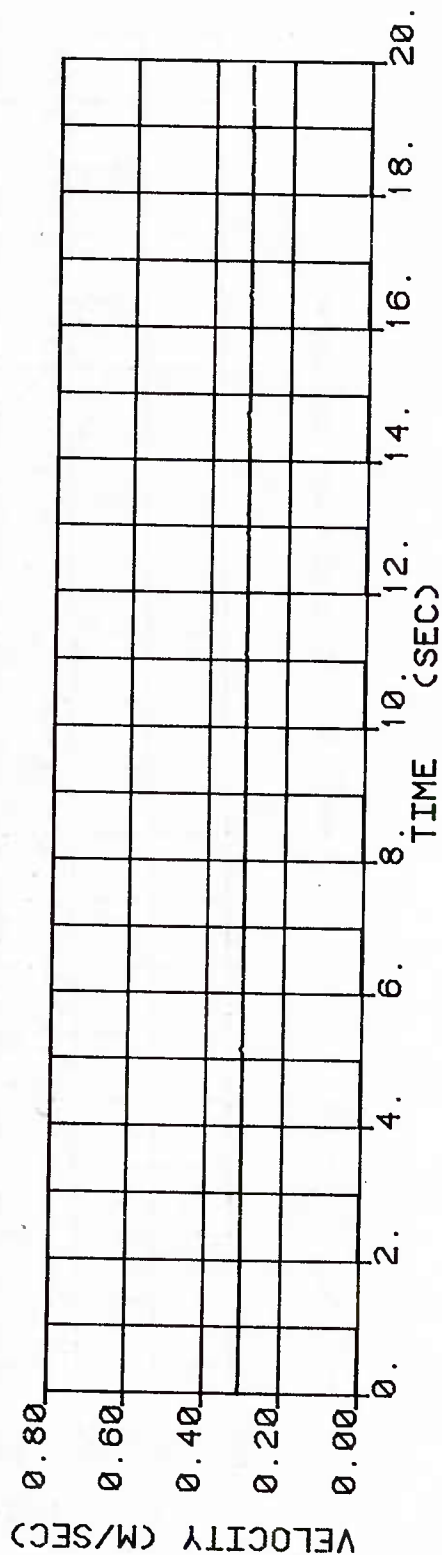
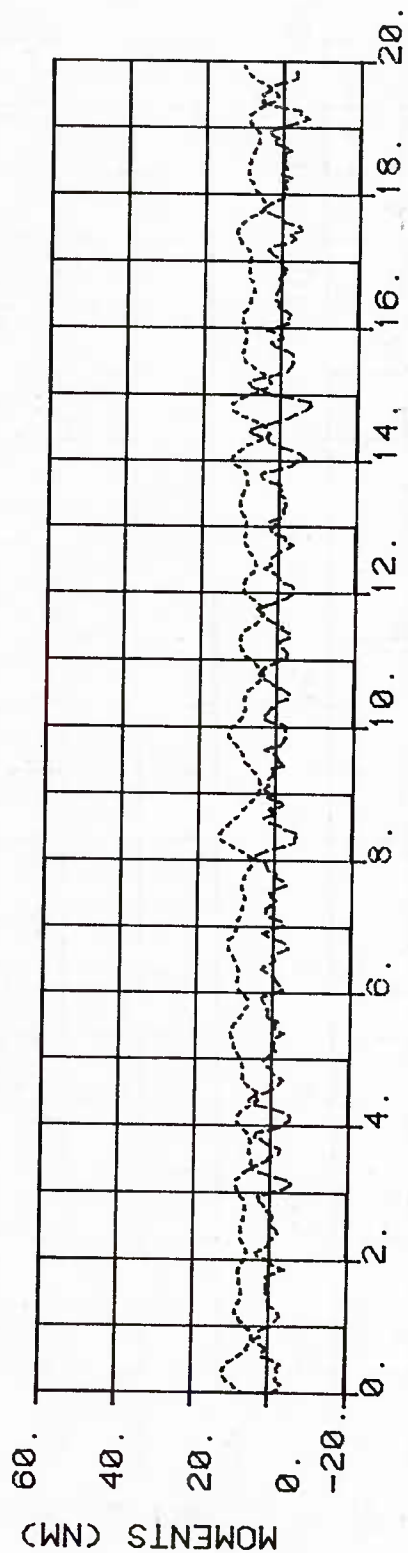
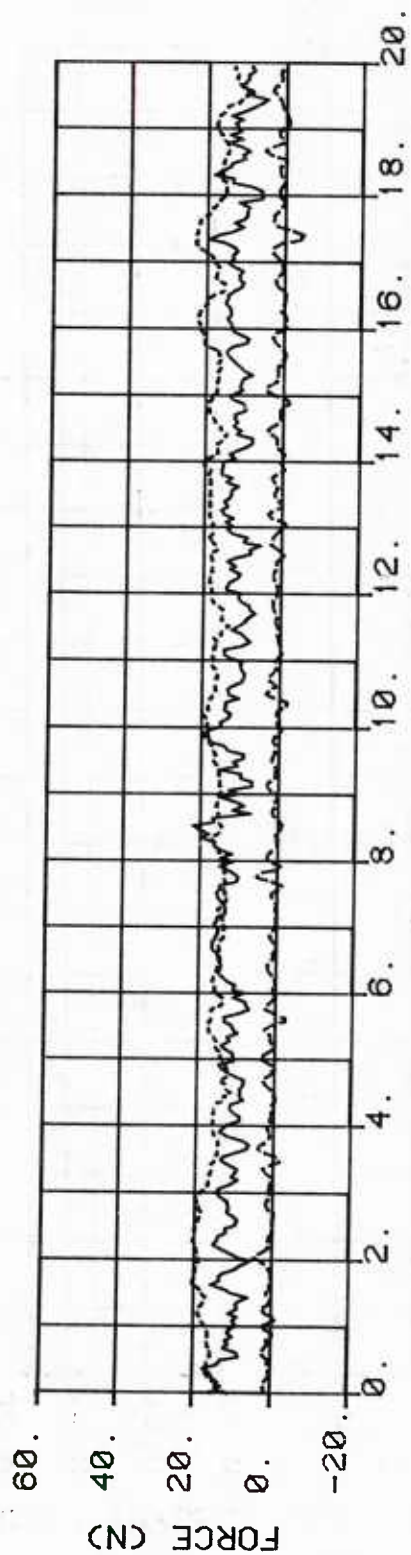
SHEET B : VEL=0.22 M/SEC, RUN 08027





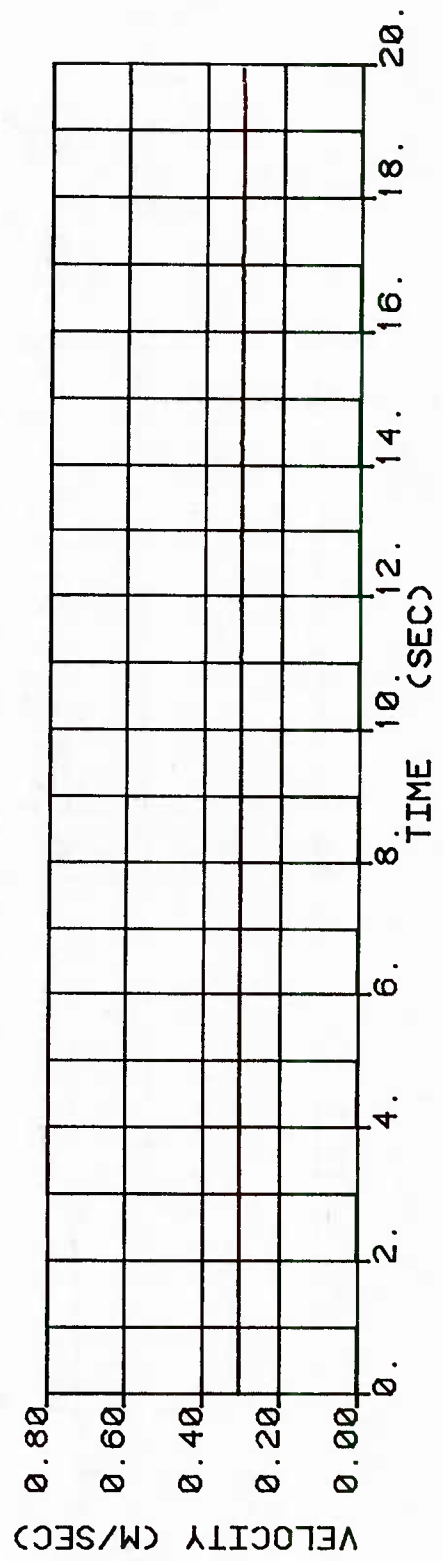
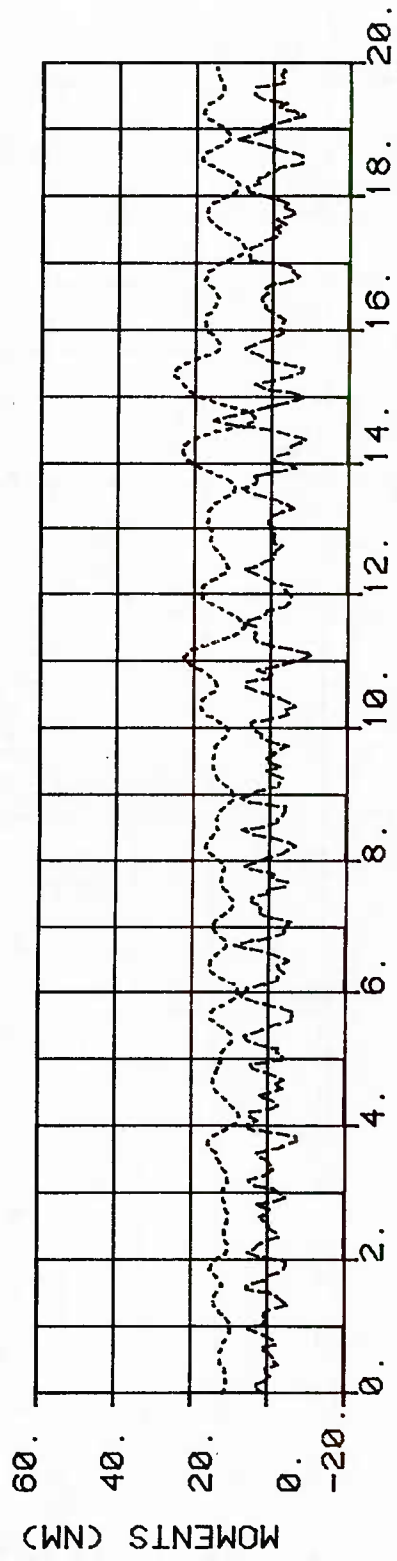
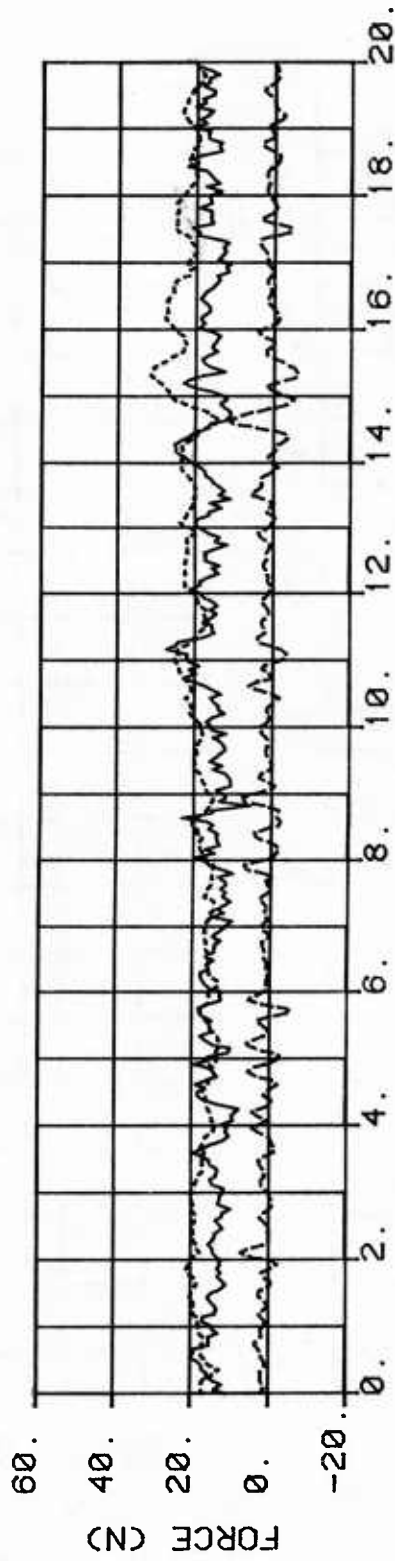
# TIME SERIES

SHEET C : VEL=0.30 M/SEC, RUN 03018



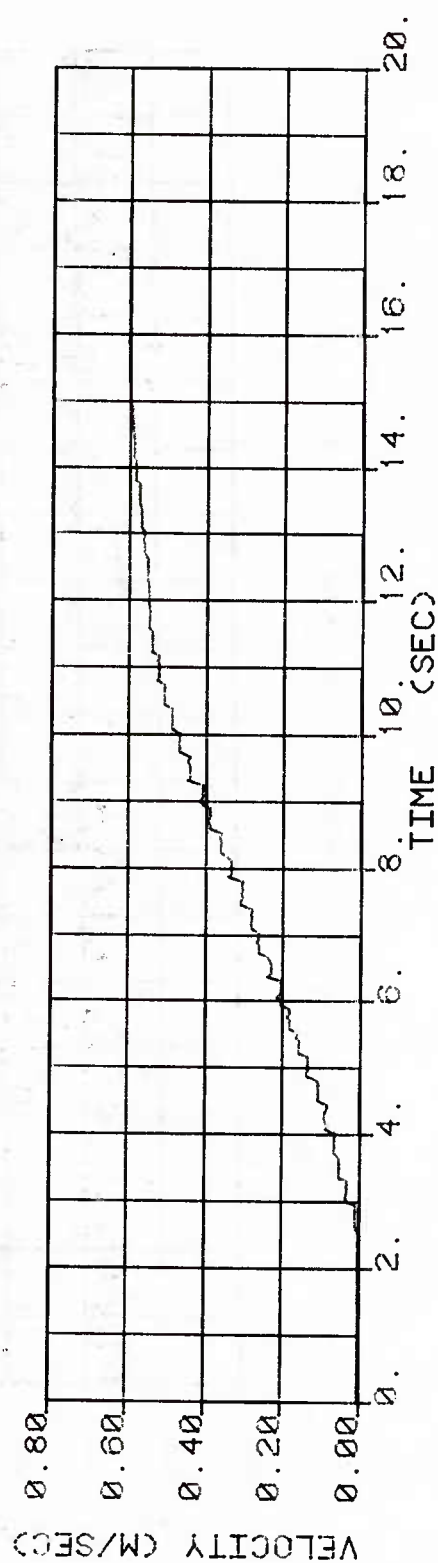
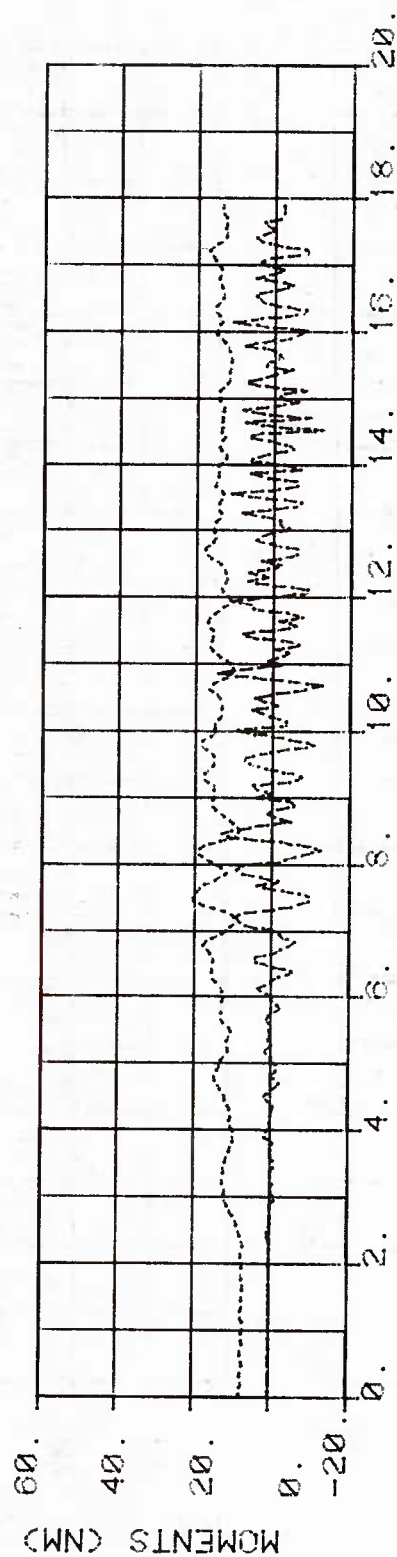
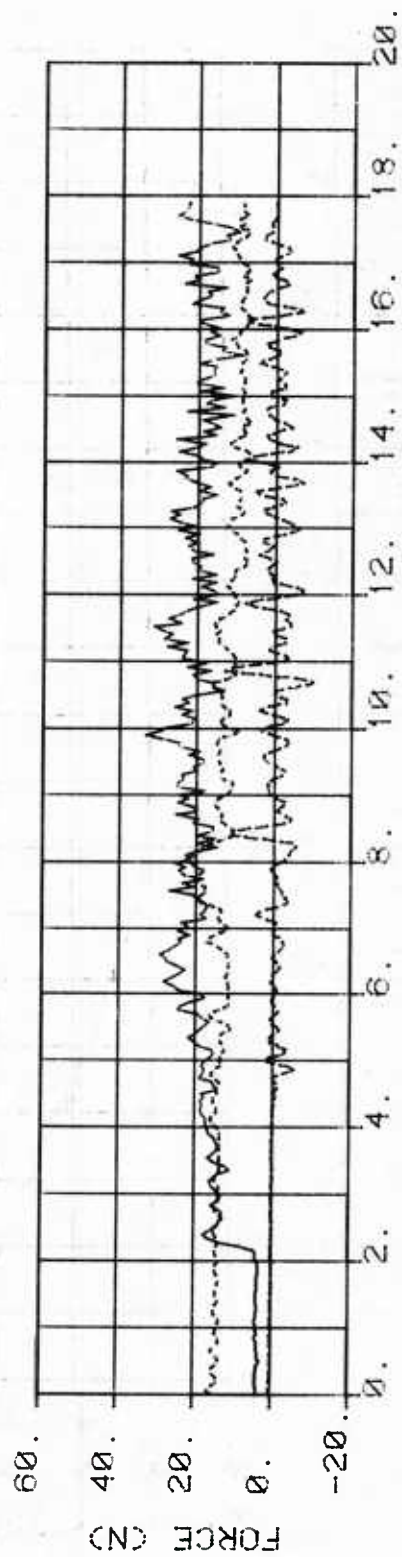
# TIME SERIES

SHEET D : VEL=0.30 M/SEC, RUN 05025



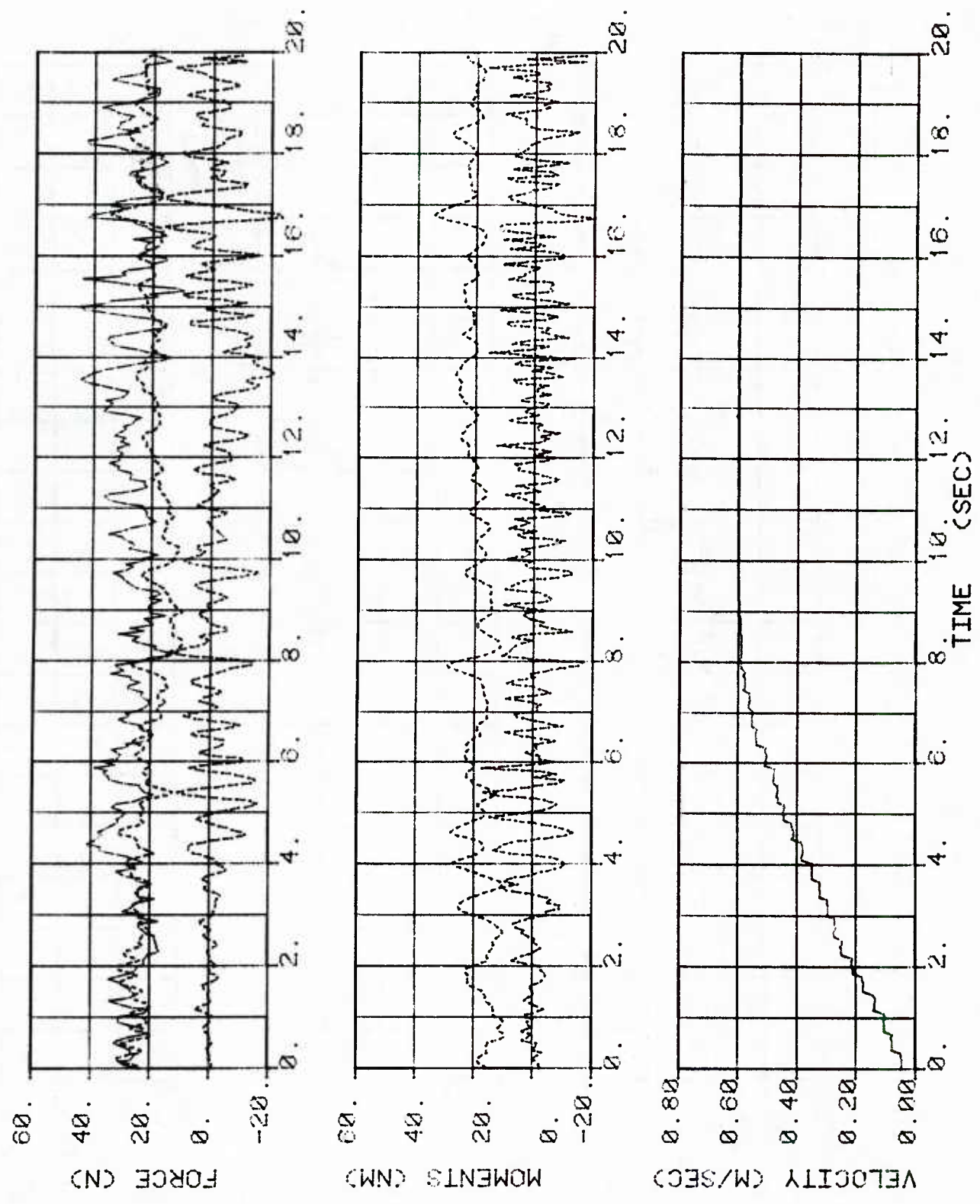
# TIME SERIES

SHEET E : VEL=0.59 M/SEC, RUN 05020



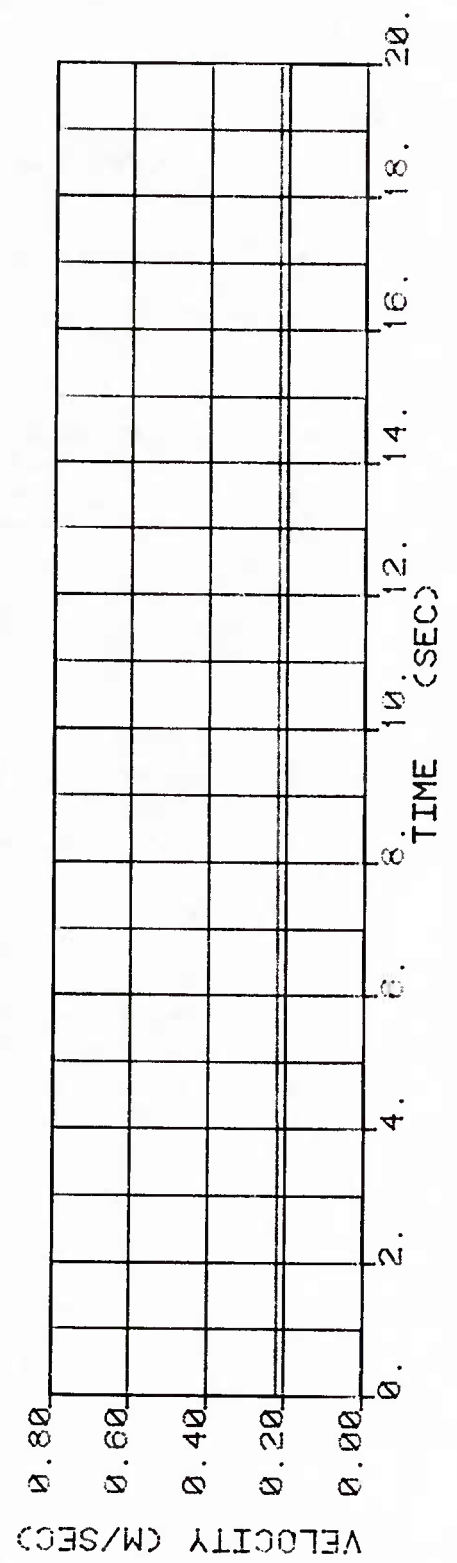
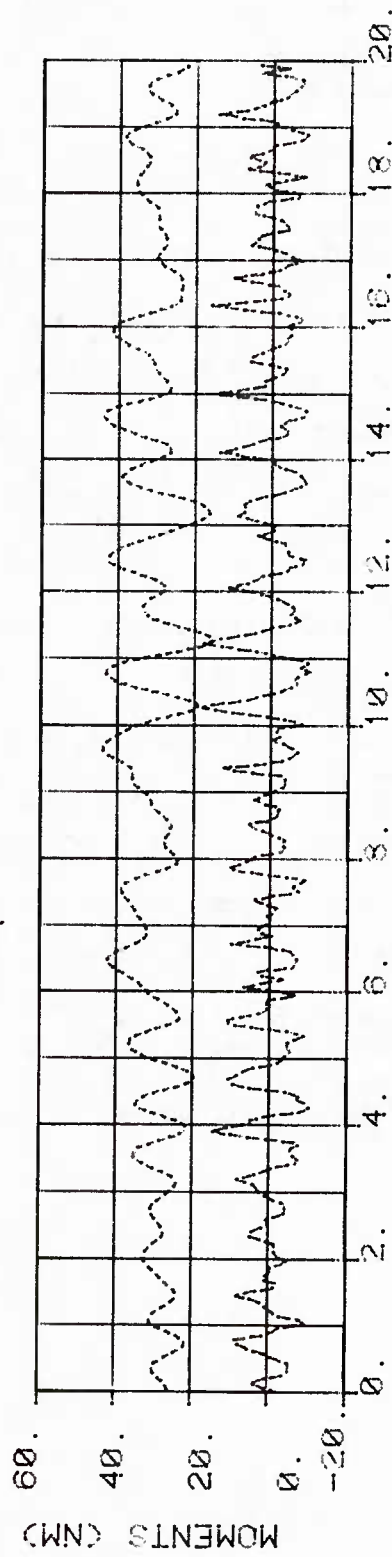
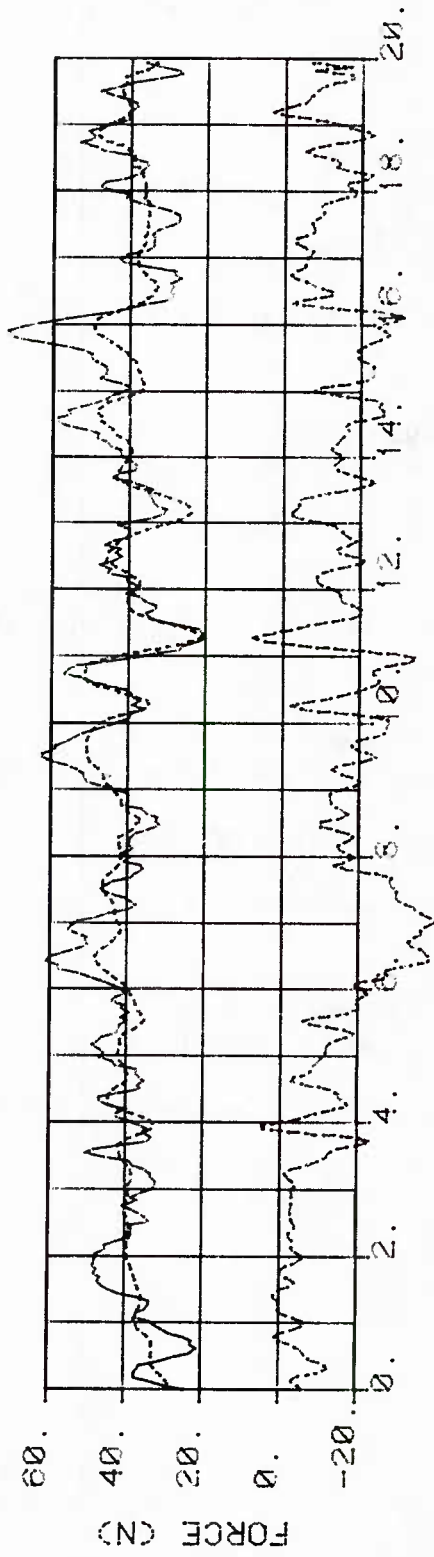
TIME SERIES

SHEET F : VEL=0.60 M/SEC, RUN 07029



# TIME SERIES

SHEET 6 : VEL=0.22 M/SEC, RUN 05022





## APPENDIX C

## Power Spectra, Cross-Spectra and Coherence-Functions

## Figure C1: Power Spectra

- a) Examples of low velocity runs,  $V_O = .03$  m/sec
- b) Runs at medium velocity,  $V_O = 0.22$  m/sec and  $V_O = 0.3$  m/sec
- c) Runs at high velocity,  $V_O = 0.6$  m/sec
- d) Runs with Sheet G,  $V_O = 0.03$  m/sec,  $V_O = 0.22$  m/sec

Figure C2: Cross Spectra and Coherence functions of  $\ddot{z}(t)$  with  $\ddot{z}(t)$ ,  $\ddot{\theta}(t)$  and  $\ddot{\theta}(t)$ 

- a) Examples of low velocity runs,  $V_O = 0.03$  m/sec
- b) Runs at medium velocity,  $V_O = .22$  m/sec and  $V_O = 0.3$  m/sec
- c) Runs at high velocity,  $V_O = 0.6$  m/sec
- d) Runs with sheet G,  $V_O = 0.03$  m/sec  $V_O = 0.22$  m/sec

Figure C3: Cross Spectra and Coherence Functions of  $\ddot{F}_x(t)$  with  $\ddot{z}(t)$ ,  $\ddot{F}_x(t)$  with  $\ddot{z}(t)$ 

- a) Examples of low velocity runs,  $V_O = 0.03$  m/sec
- b) Runs at medium velocity,  $V_O = 0.22$  m/sec and  $V_O = 0.3$  m/sec
- c) Runs at high velocity,  $V_O = 0.6$  m/sec
- d) Runs with sheet G,  $V_O = 0.03$  m/sec and  $V_O = 0.22$  m/sec

Figure C4: Cross-Spectra and Coherence Functions of  $\ddot{F}_x(t)$  with  $\ddot{\theta}(t)$  and  $\ddot{F}_x(t)$  with  $\ddot{\theta}(t)$ 

- a) Examples of low velocity runs,  $V_O = 0.03$  m/sec
- b) Runs at medium velocity,  $V_O = 0.22$  m/sec and  $V_O = 0.3$  m/sec
- c) Runs at high velocity,  $V_O = 0.6$  m/sec
- d) Runs with sheet G,  $V_O = 0.03$  m/sec and  $V_O = 0.22$  m/sec

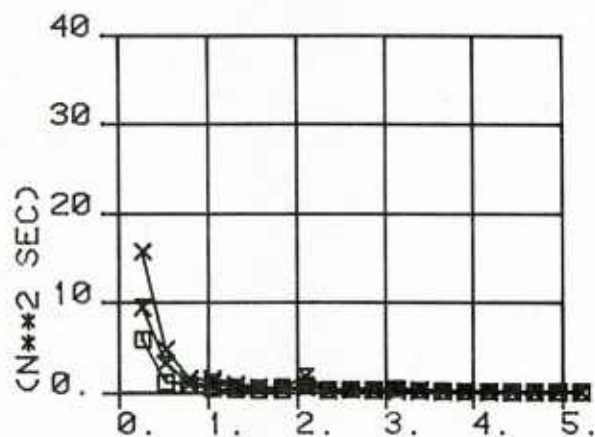
Figure C1: Power Spectra



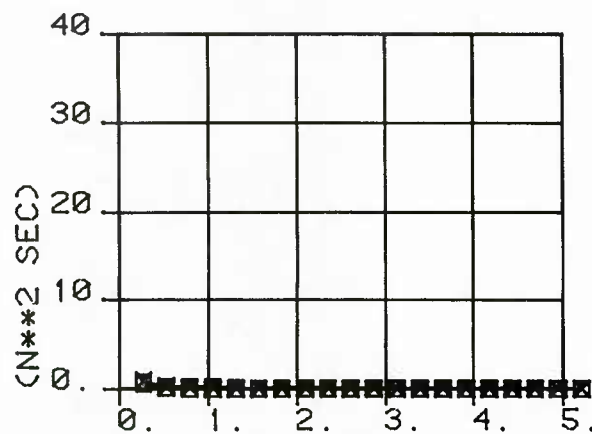
## POWER-SPECTRUM

VEL = .03 M/SEC : SHEET A  $\square$  , SHEET D  $\times$   
 SHEET F  $\times$

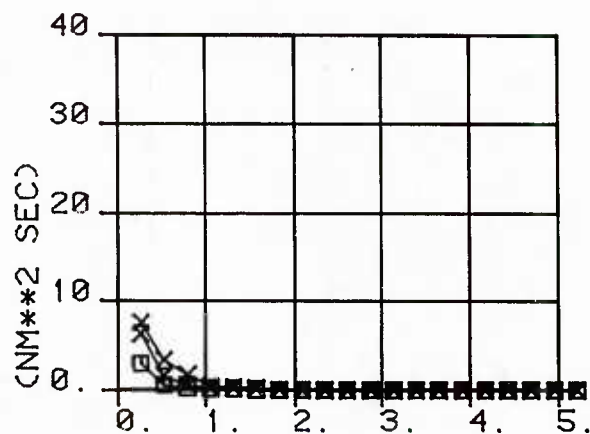
## HORIZONTAL FORCE



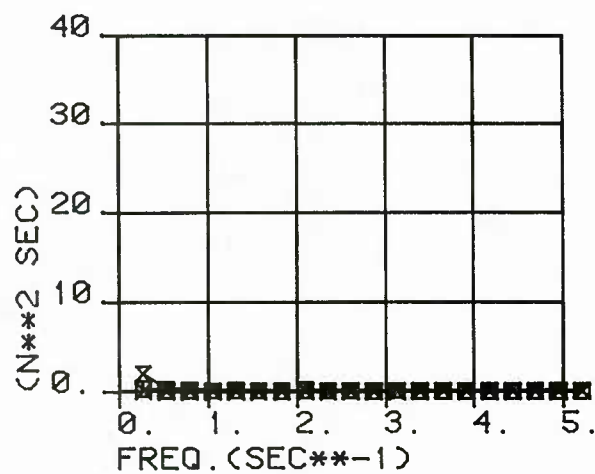
## DRAFT



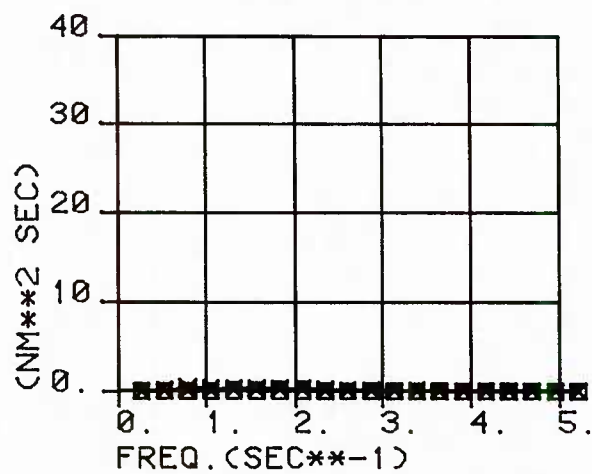
## PITCH



## VERT. ACCELERATION



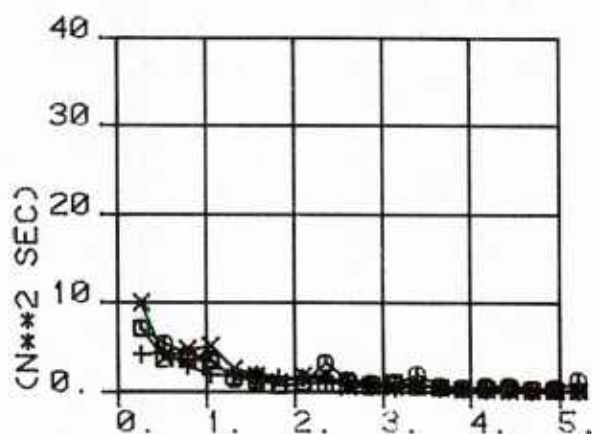
## ANG. ACCELERATION



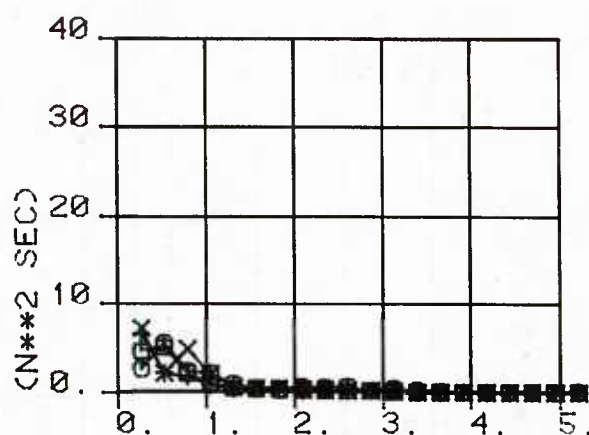
## POWER-SPECTRUM

VEL = .22 M/SEC : SHEET A  $\square$  , SHEET B  $\circ$   
 VEL = .30 M/SEC : SHEET C  $+$  , SHEET D  $\times$

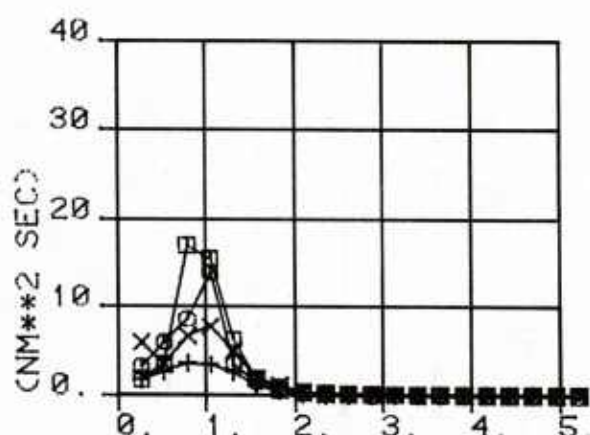
## HORIZONTAL FORCE



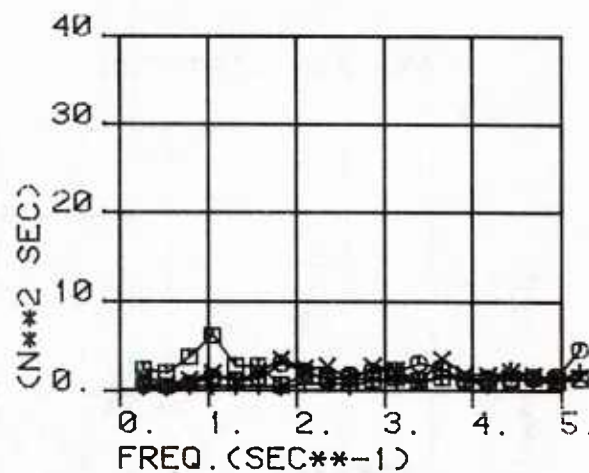
## DRAFT



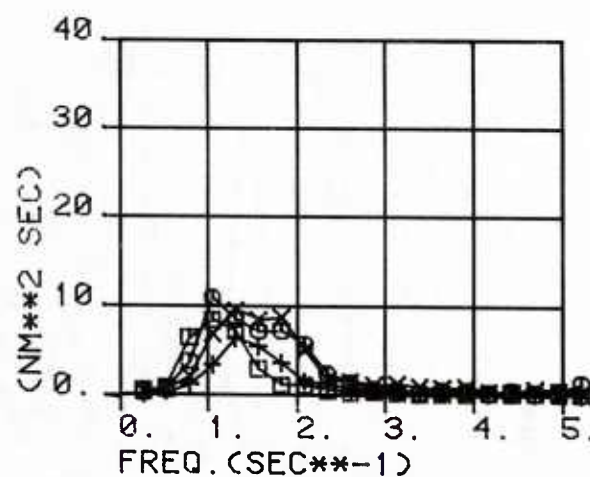
## PITCH



## VERT. ACCELERATION



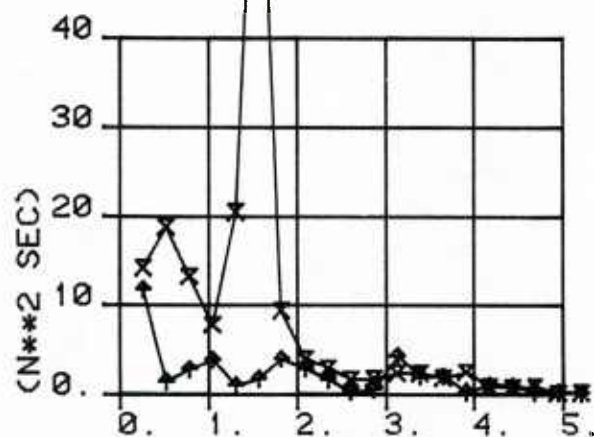
## ANG. ACCELERATION



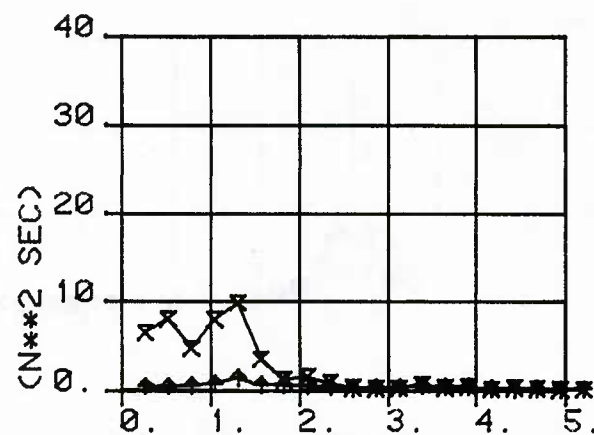
## POWER-SPECTRUM

VEL = .60 M/SEC : SHEET E  $\uparrow$   
 SHEET F  $\times$

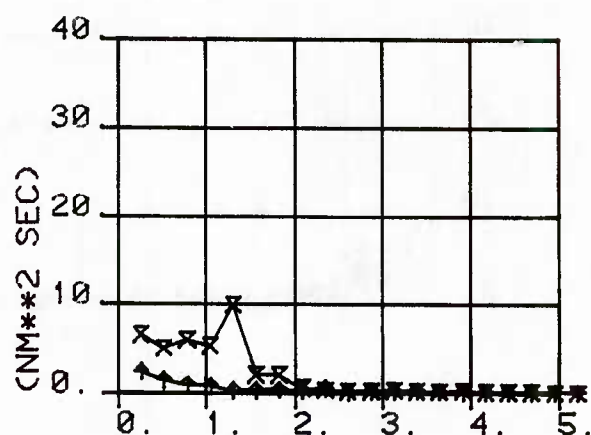
## HORIZONTAL FORCE



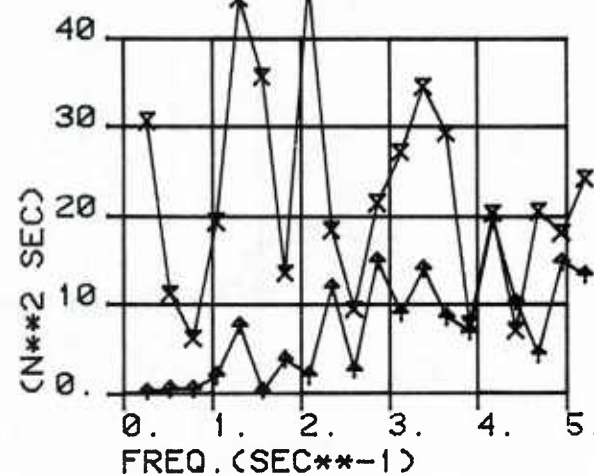
## DRAFT



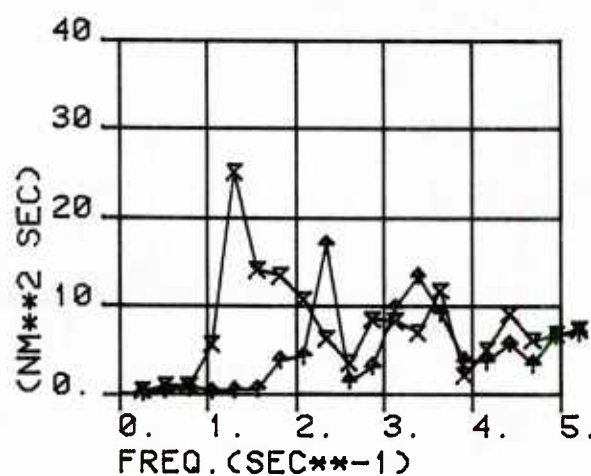
## PITCH



## VERT. ACCELERATION



## ANG. ACCELERATION

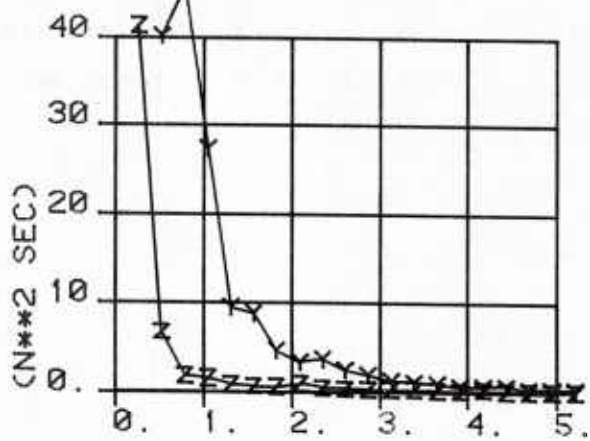


## POWER-SPECTRUM

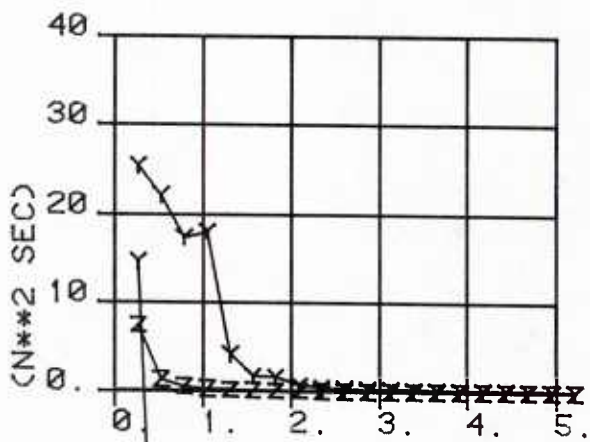
VEL = .03 M/SEC : SHEET G Z

VEL = .22 M/SEC : SHEET G Y

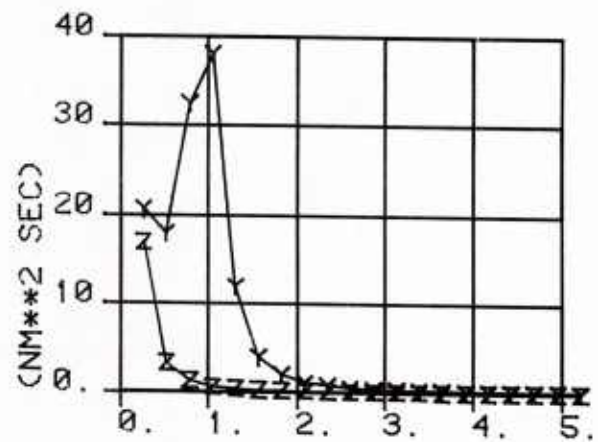
## HORIZONTAL FORCE



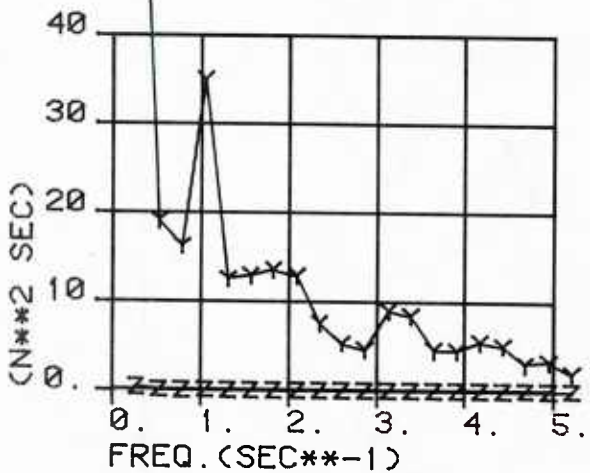
## DRAFT



## PITCH



## VERT. ACCELERATION



## ANG. ACCELERATION

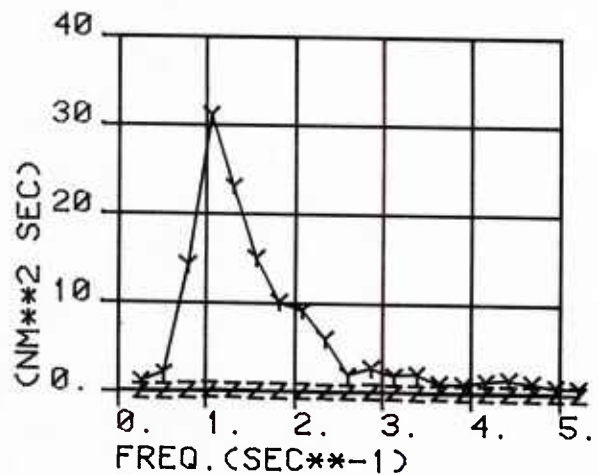
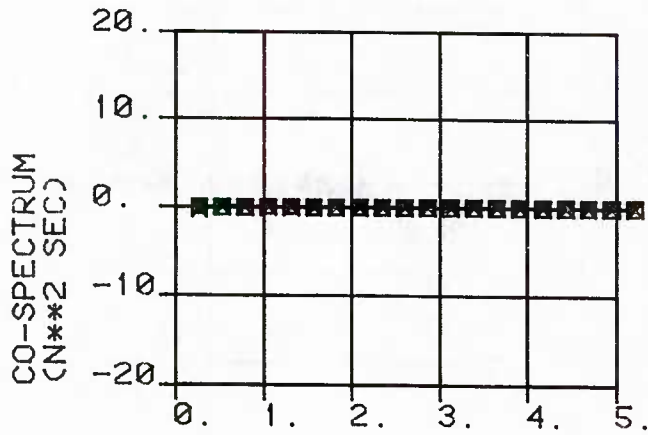


Figure C2: Cross-Spectra and Coherence Functions of Draft  $z(t)$  with Vertical Acceleration  $\ddot{z}(t)$ , and Pitch  $\theta(t)$  with Angular Acceleration  $\ddot{\theta}(t)$

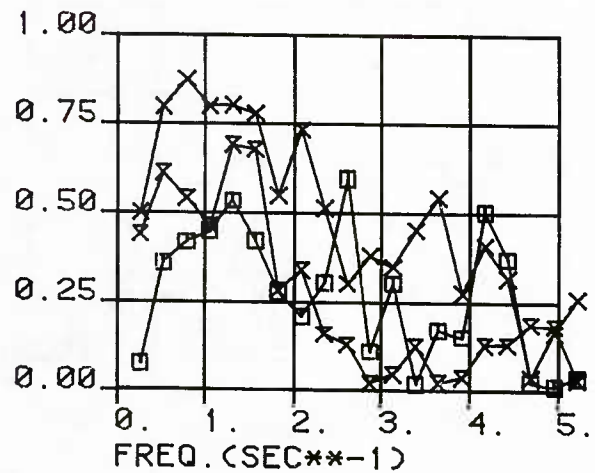
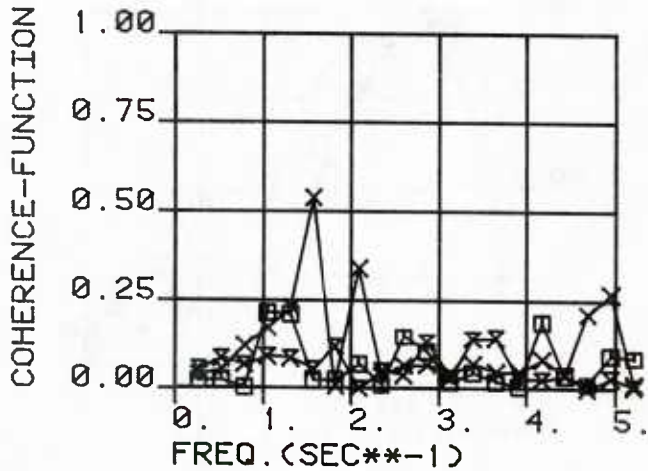
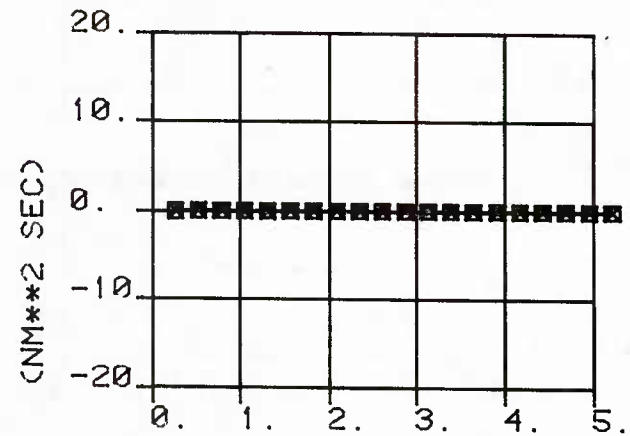
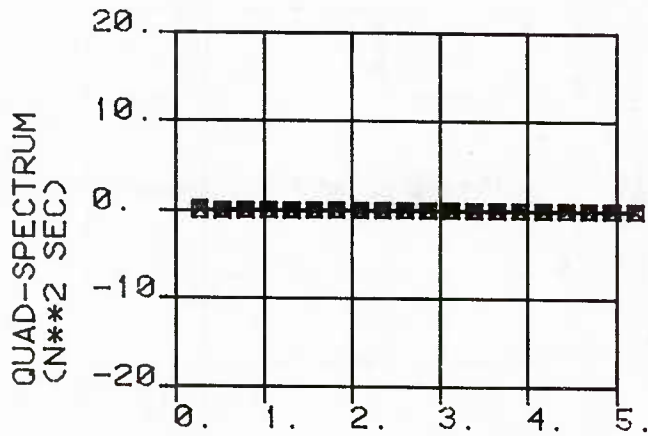
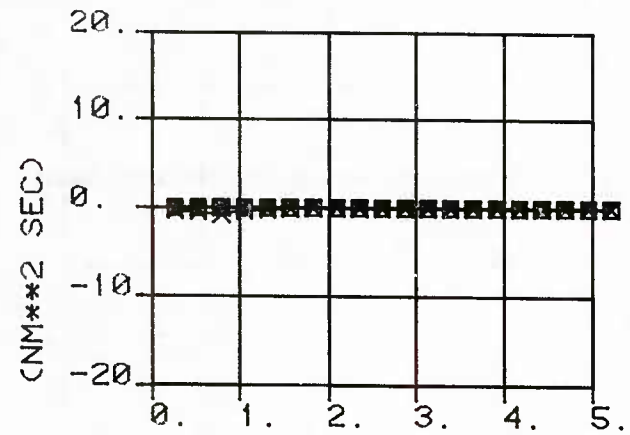
## CROSS-SPECTRUM AND COHERENCE-FUNCTION

VEL = .03 M/SEC : SHEET A  $\square$ , SHEET D  $\times$   
 SHEET F  $\times$

DRAFT WITH VERT. ACC



PITCH WITH ANG. ACC

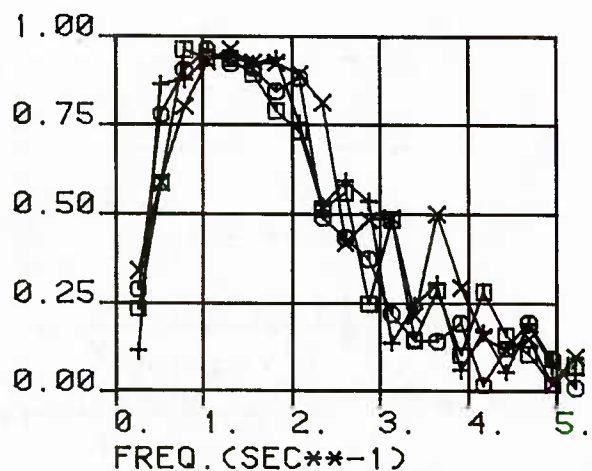
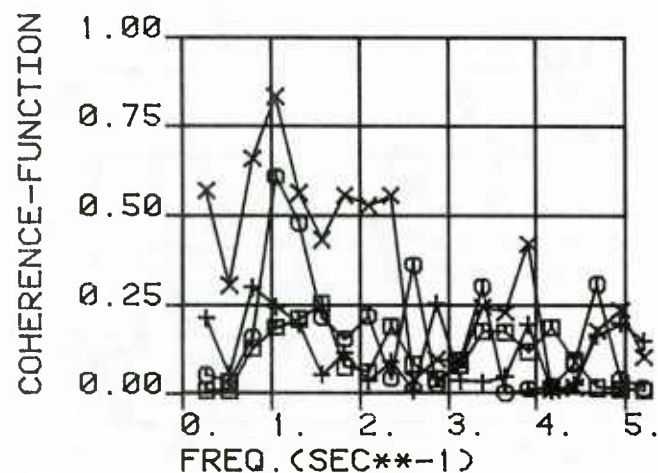
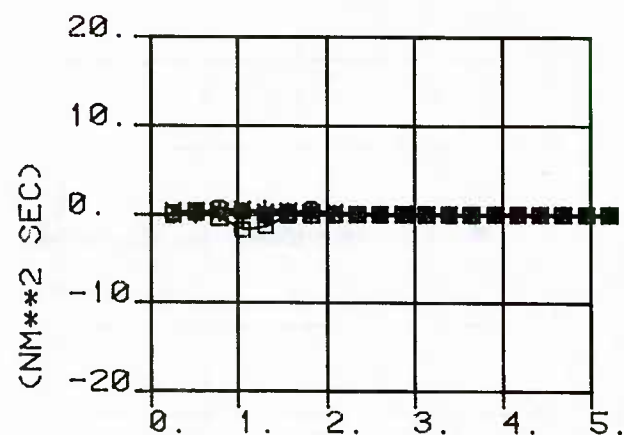
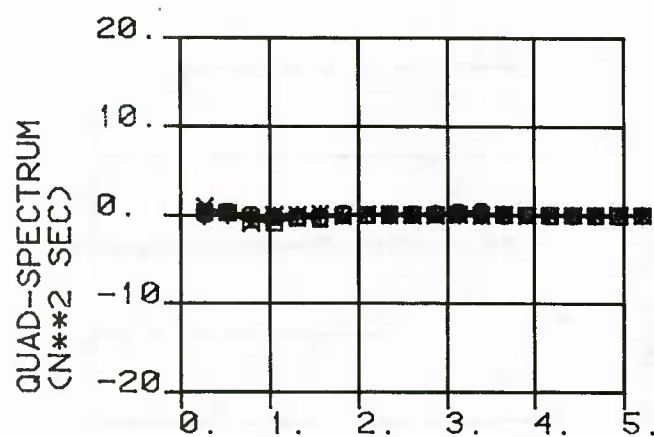
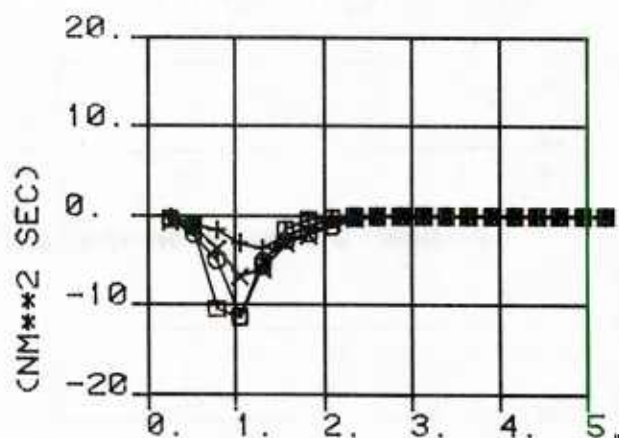
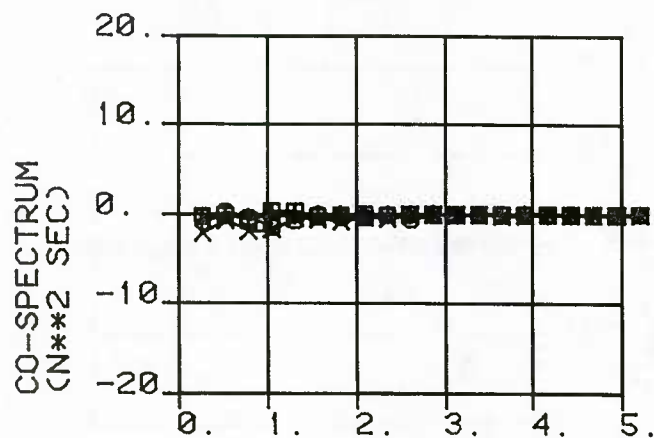


## CROSS-SPECTRUM AND COHERENCE-FUNCTION

VEL = .22 M/SEC : SHEET A  $\square$  , SHEET B  $\circ$ VEL = .30 M/SEC : SHEET C  $+$  , SHEET D  $\times$ 

DRAFT WITH VERT. ACC

PITCH WITH ANG. ACC

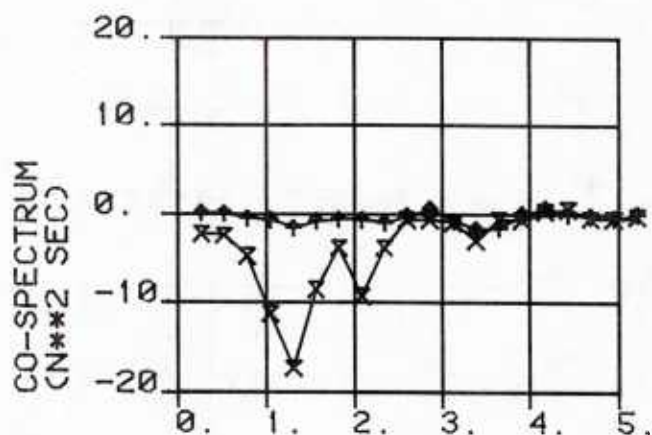




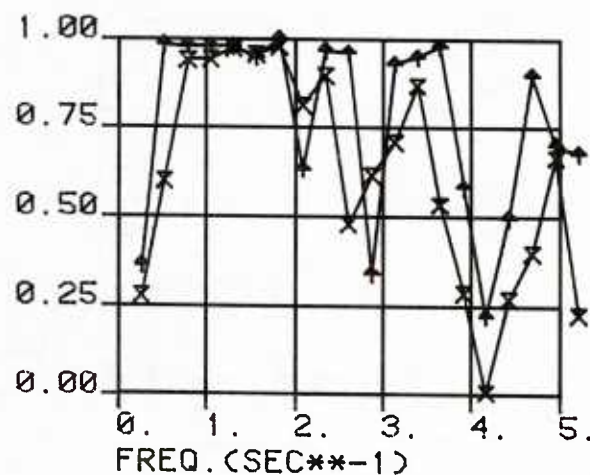
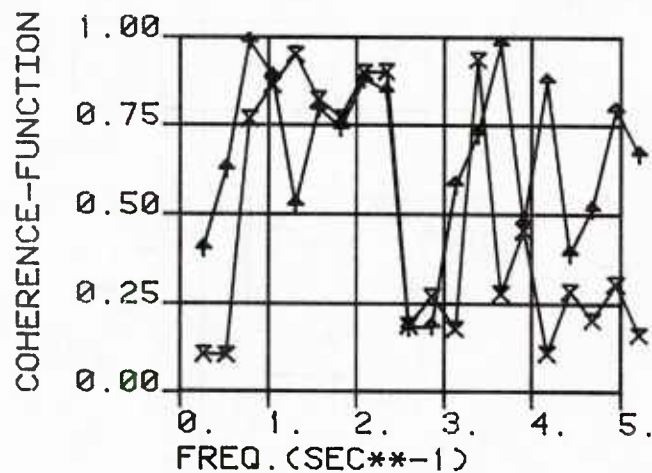
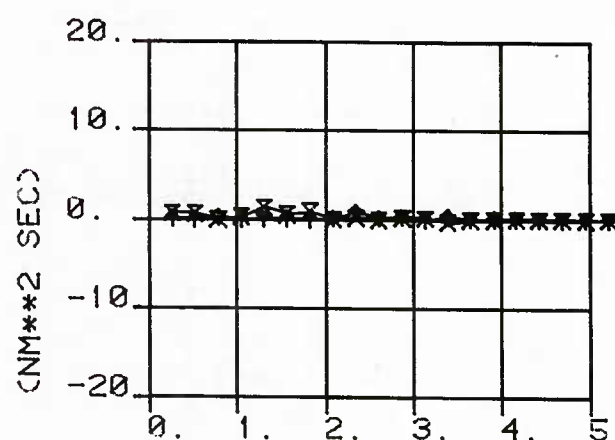
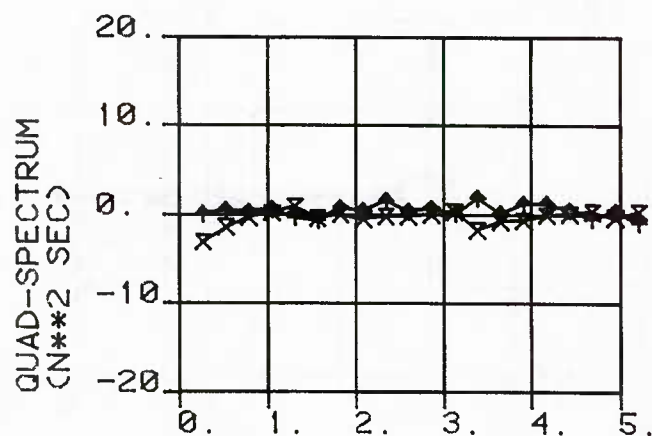
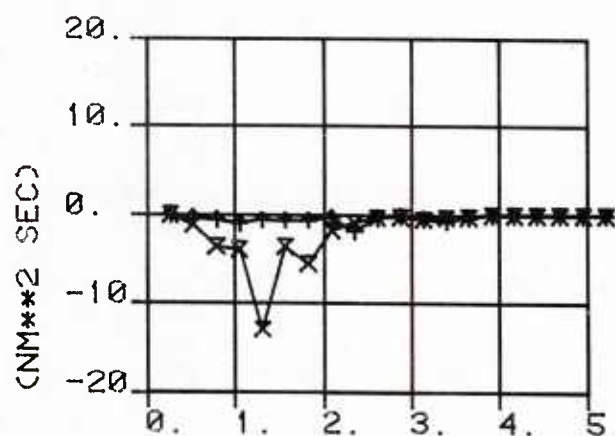
## CROSS-SPECTRUM AND COHERENCE-FUNCTION

VEL = .60 M/SEC : SHEET E  $\uparrow$ SHEET F  $\times$ 

DRAFT WITH VERT. ACC



PITCH WITH ANG. ACC



## CROSS-SPECTRUM AND COHERENCE-FUNCTION

VEL = .03 M/SEC : SHEET G Z

VEL = .22 M/SEC : SHEET G Y

DRAFT WITH VERT.ACC

PITCH WITH ANG.ACC

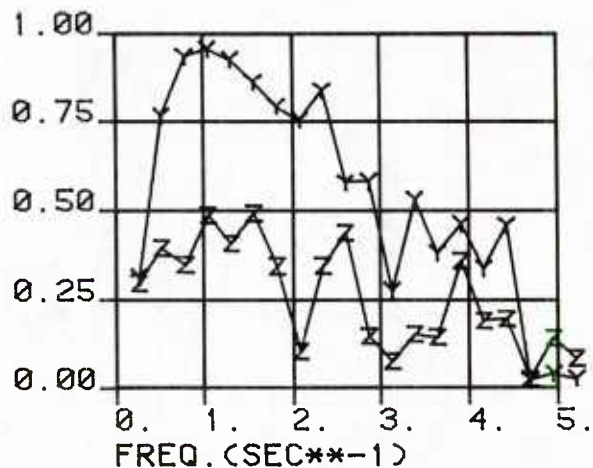
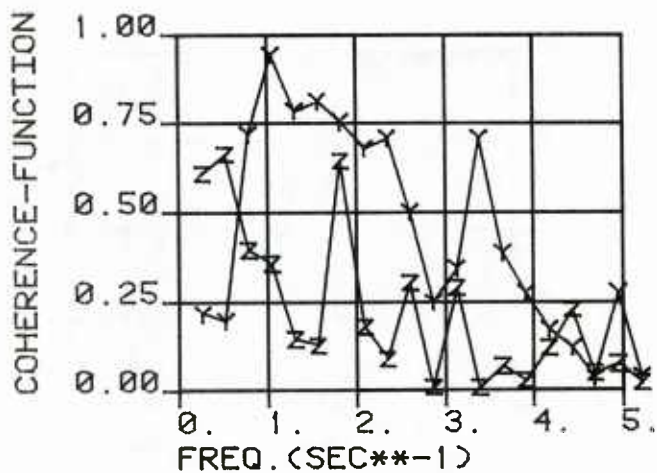
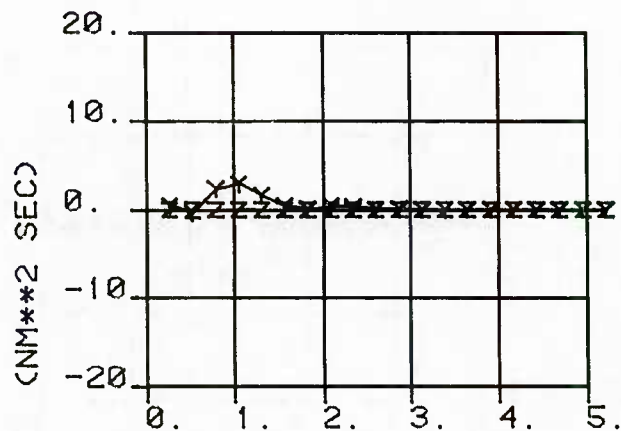
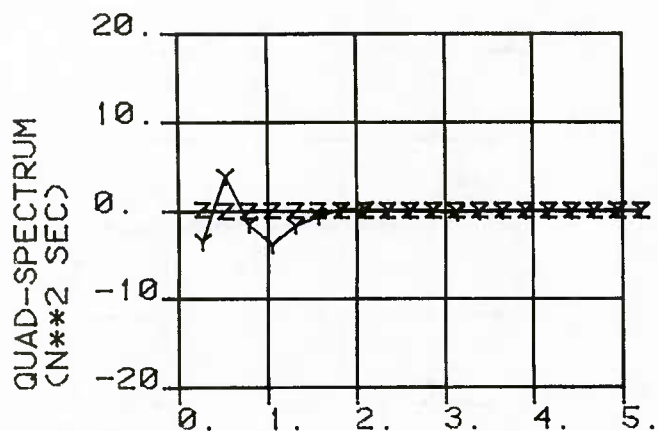
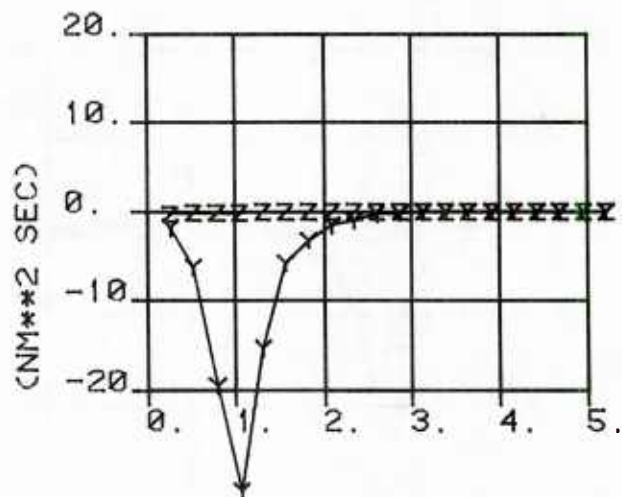
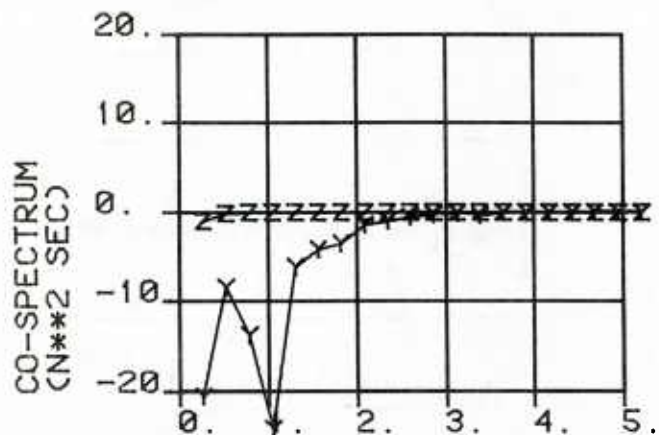
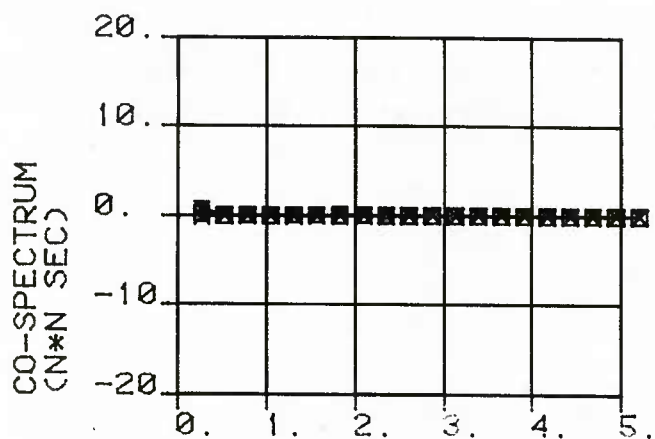


Figure C3: Cross-Spectra and Coherence Functions of  $F_x(t)$  with draft  $z(t)$ , and  $F_x(t)$  with Vertical Acceleration  $\ddot{z}(t)$

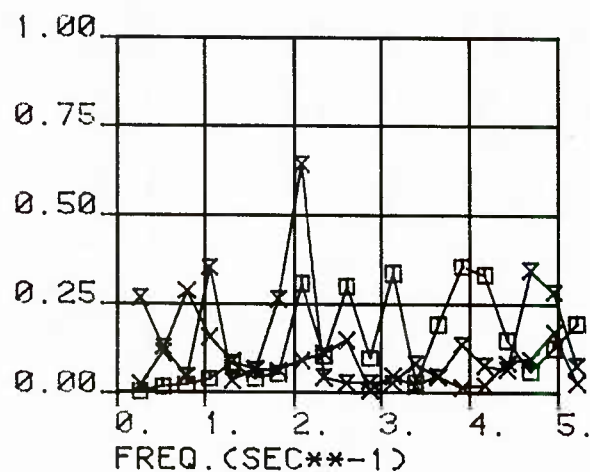
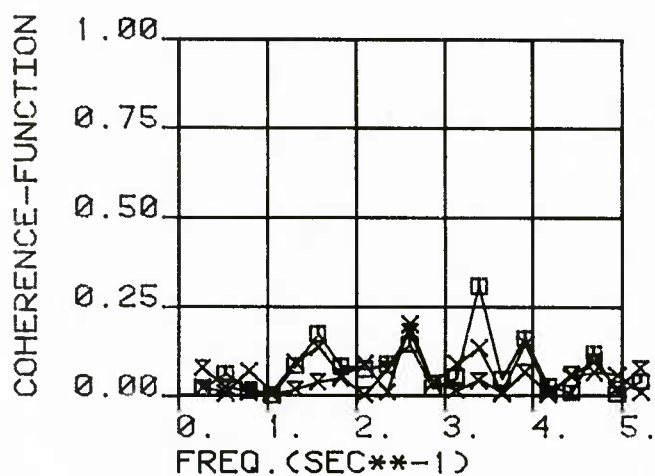
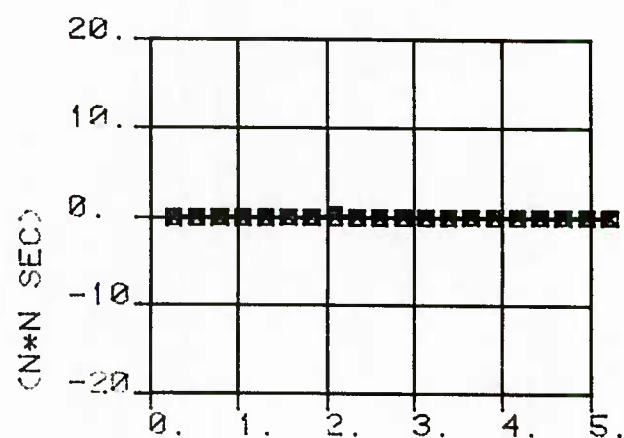
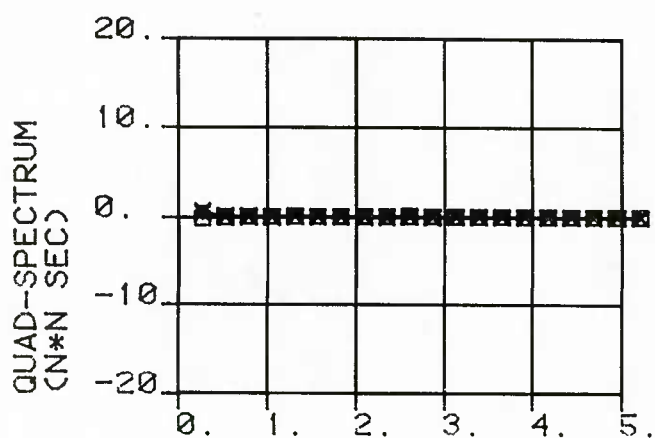
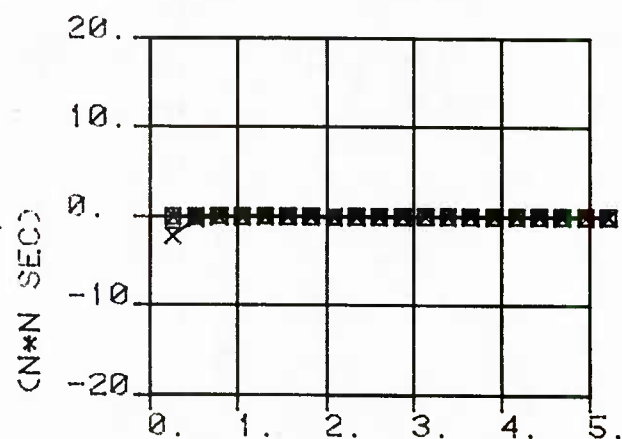
## CROSS-SPECTRUM AND COHERENCE-FUNCTION

VEL = .03 M/SEC : SHEET A  $\square$  , SHEET D  $\times$   
 SHEET F  $\times$

FX WITH DRAFT



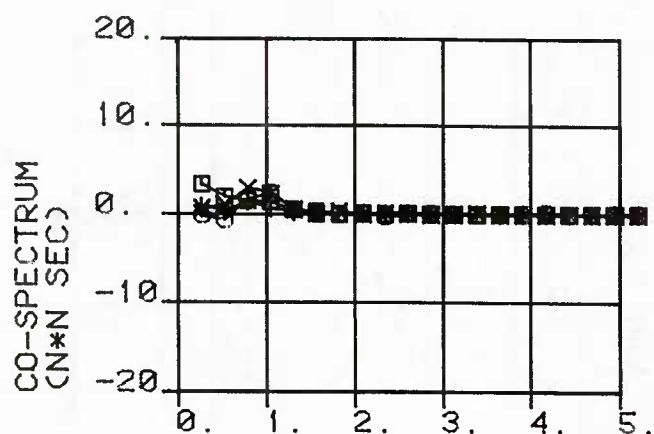
FX WITH VERT. ACC



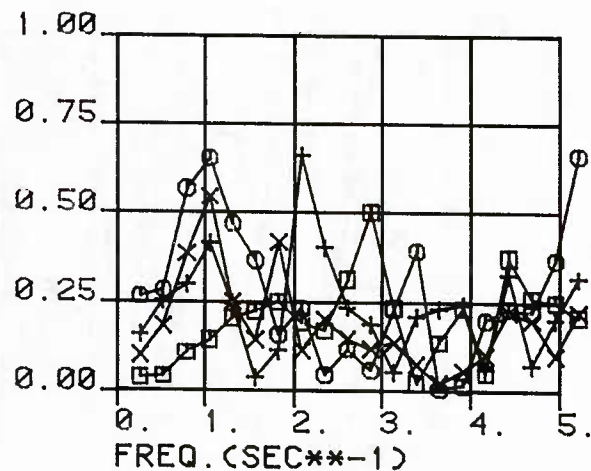
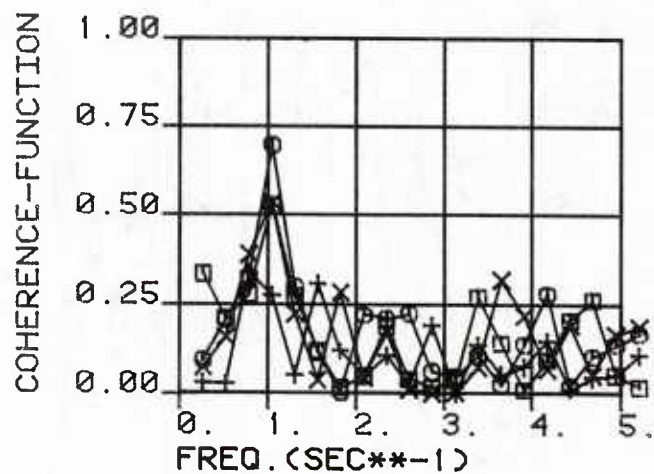
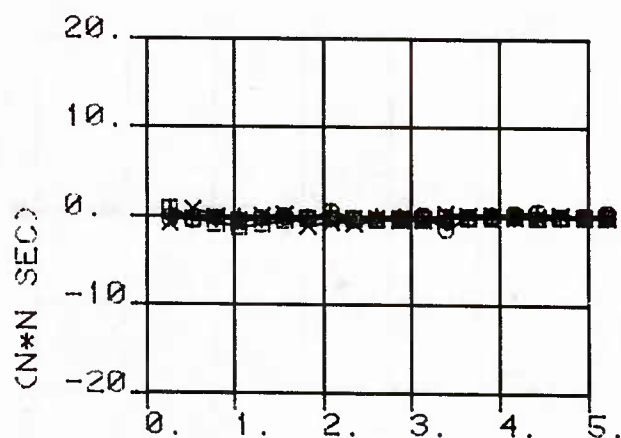
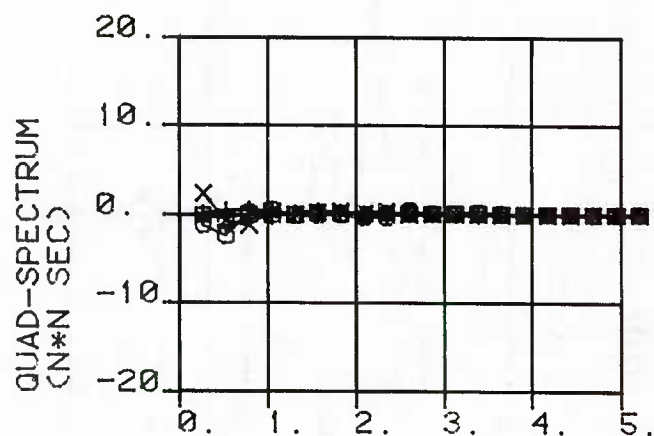
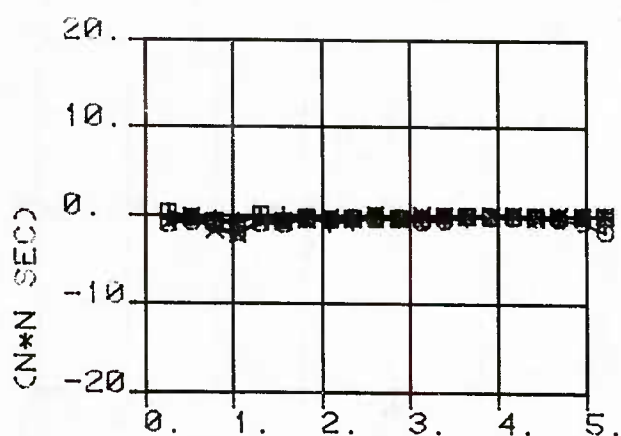
## CROSS-SPECTRUM AND COHERENCE-FUNCTION

VEL = .22 M/SEC : SHEET A  $\square$  , SHEET B  $\circ$ VEL = .30 M/SEC : SHEET C  $+$  , SHEET D  $\times$ 

FX WITH DRAFT



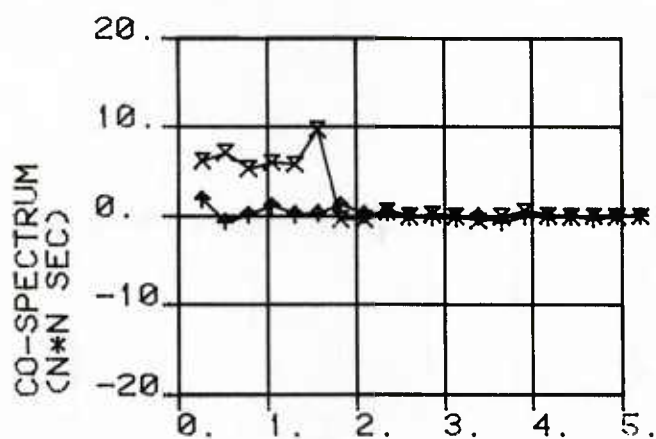
FX WITH VERT. ACC



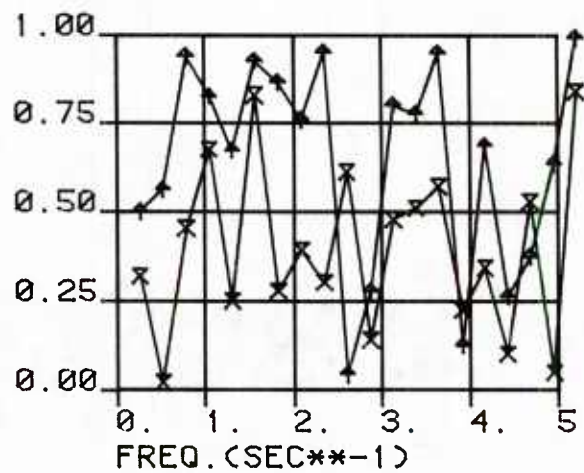
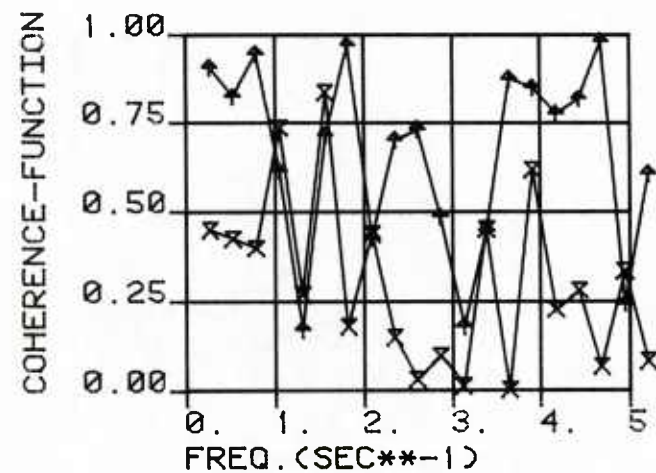
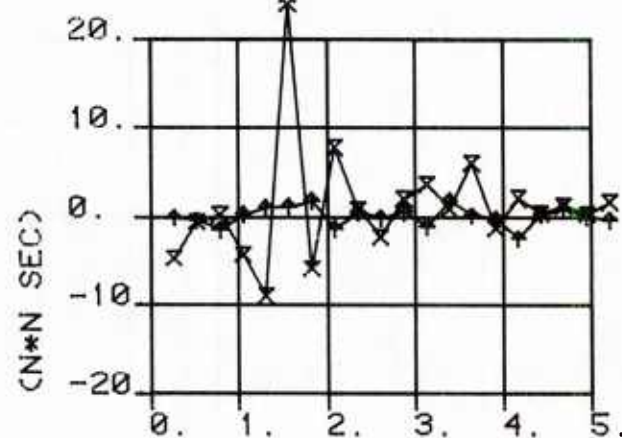
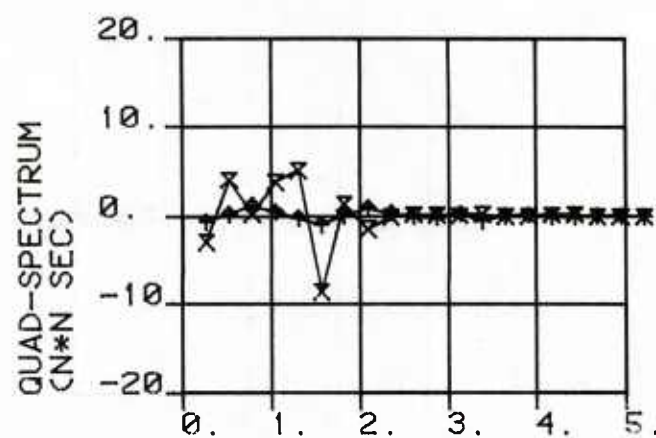
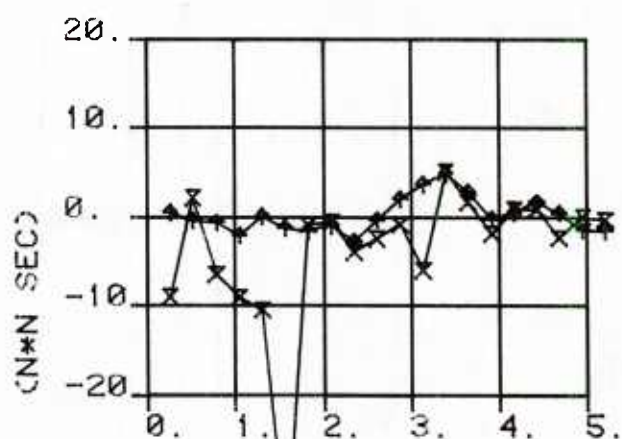
## CROSS-SPECTRUM AND COHERENCE-FUNCTION

VEL = .60 M/SEC : SHEET E  $\uparrow$ SHEET F  $\times$ 

FX WITH DRAFT



FX WITH VERT. ACC



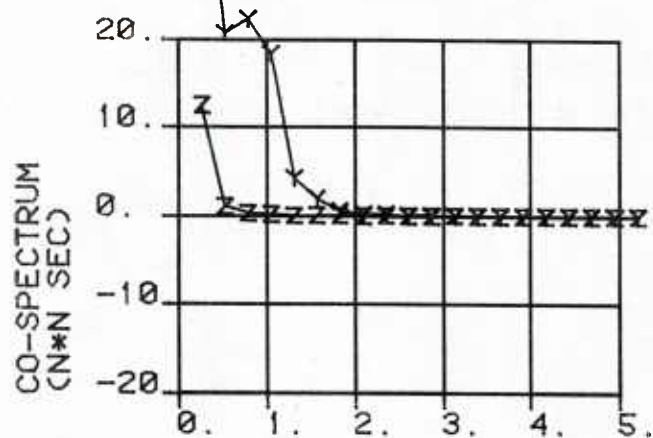


## CROSS-SPECTRUM AND COHERENCE-FUNCTION

VEL = .03 M/SEC : SHEET G Z

VEL = .22 M/SEC : SHEET G Y

FX WITH DRAFT



FX WITH VERT. ACC

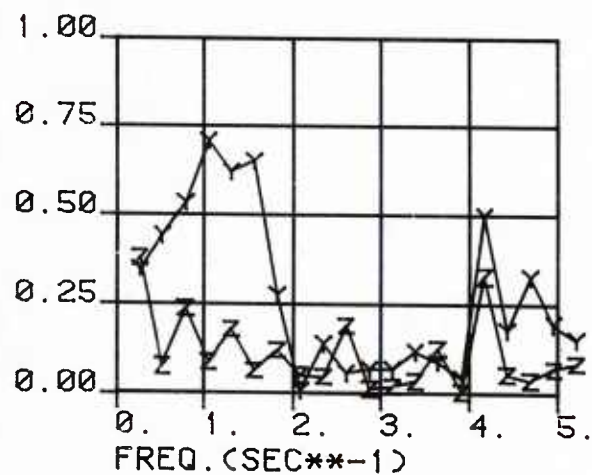
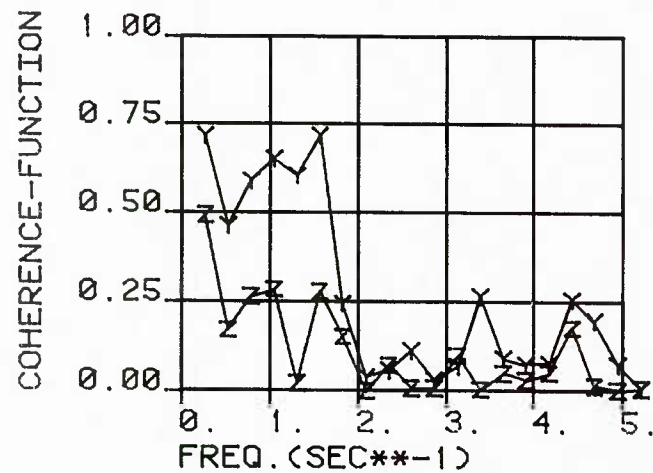
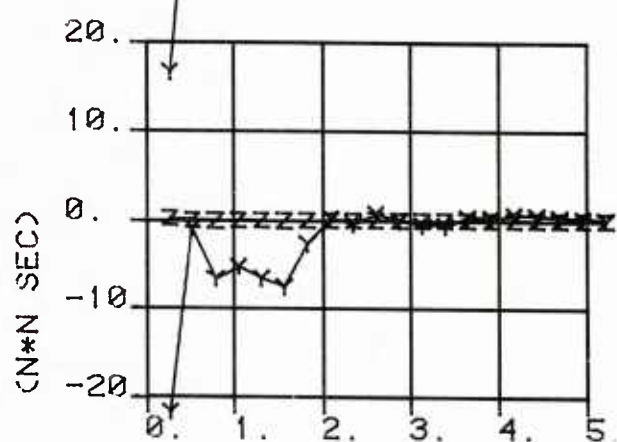
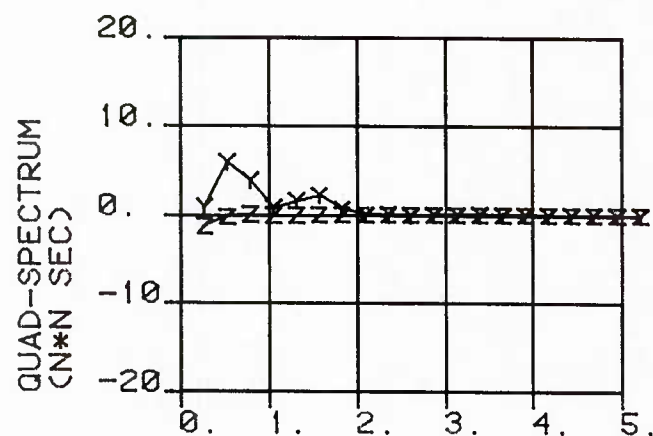
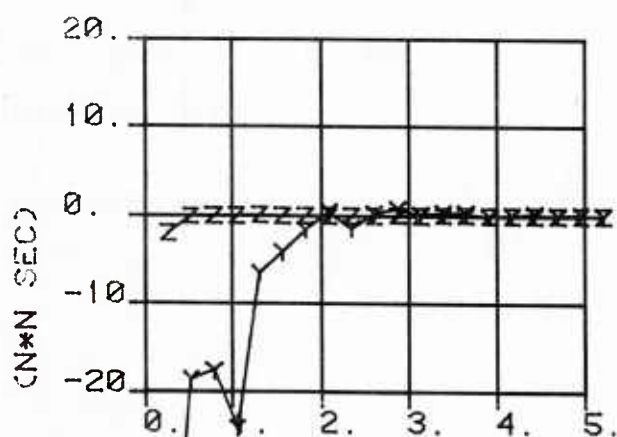


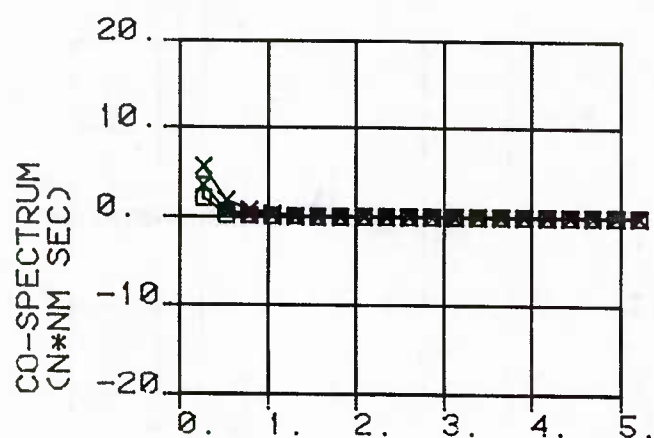


Figure C4: Cross-Spectra and Coherence Functions of  $F_x(t)$  with Pitch  $\theta(t)$  and  $F_x(t)$  with Angular Acceleration,  $\ddot{\theta}(t)$

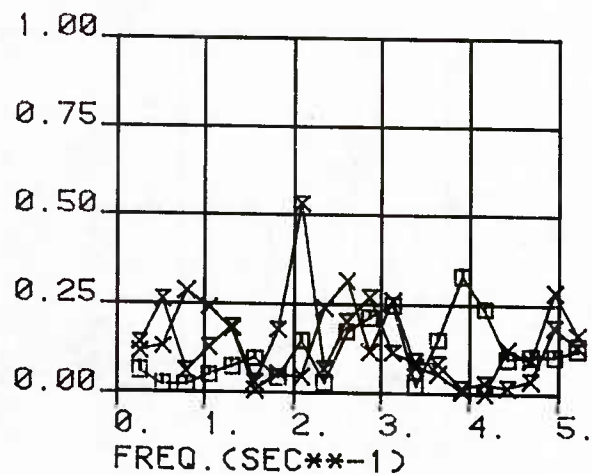
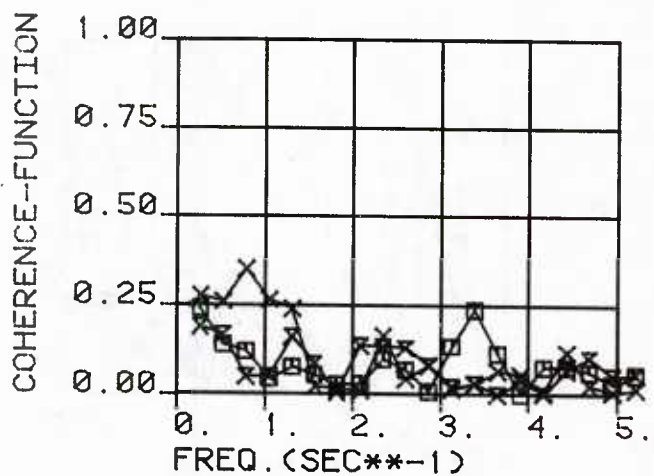
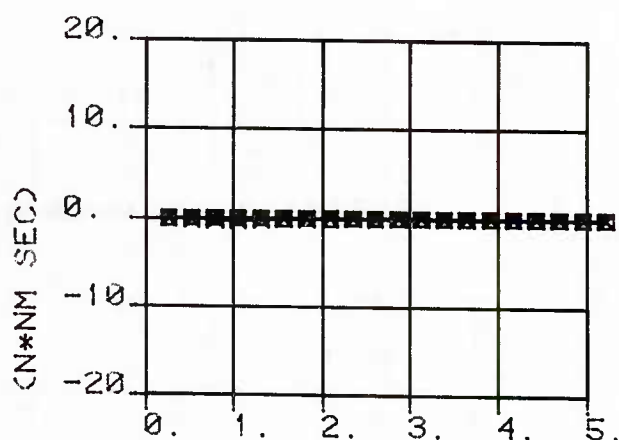
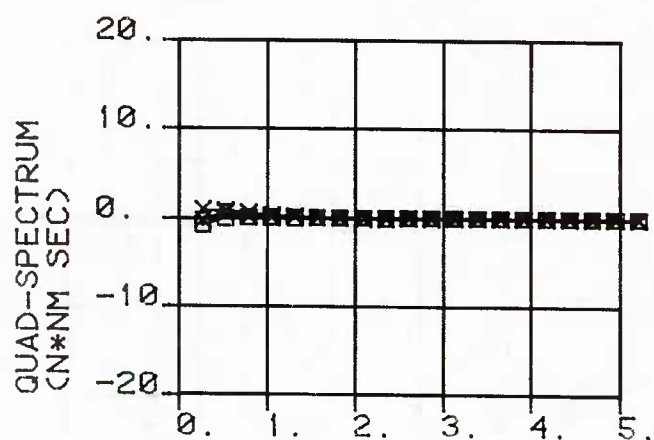
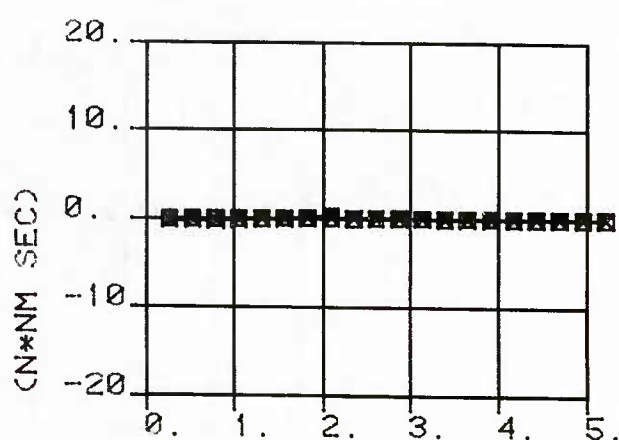
## CROSS-SPECTRUM AND COHERENCE-FUNCTION

VEL = .03 M/SEC : SHEET A  $\square$ , SHEET D  $\times$   
 SHEET F  $\times$

FX WITH PITCH



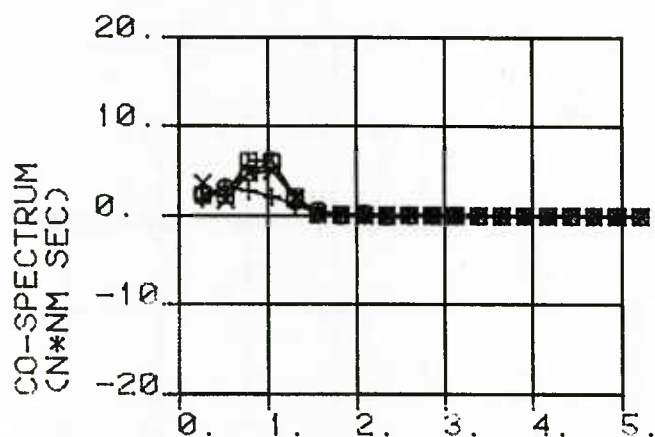
FX WITH ANG. ACC



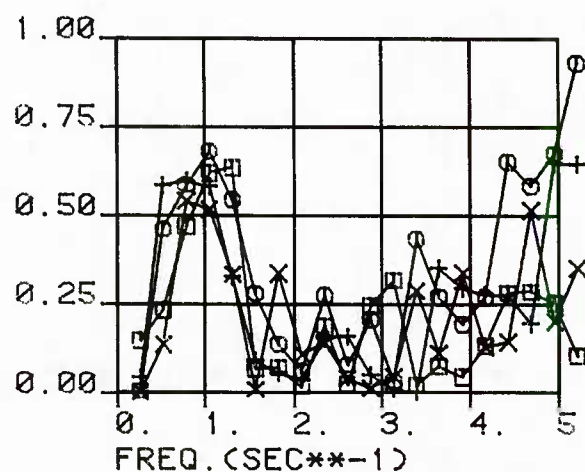
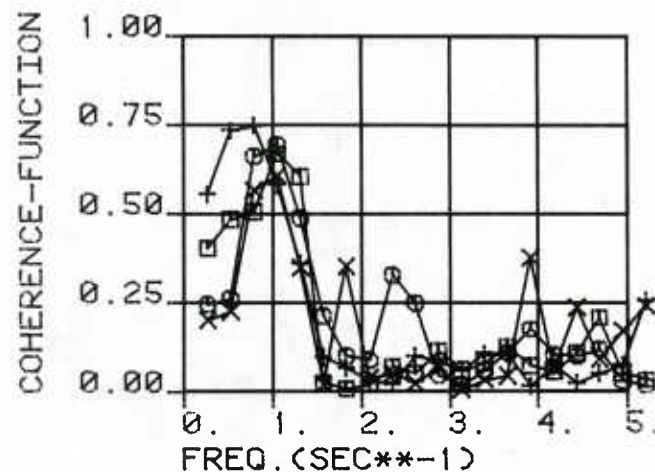
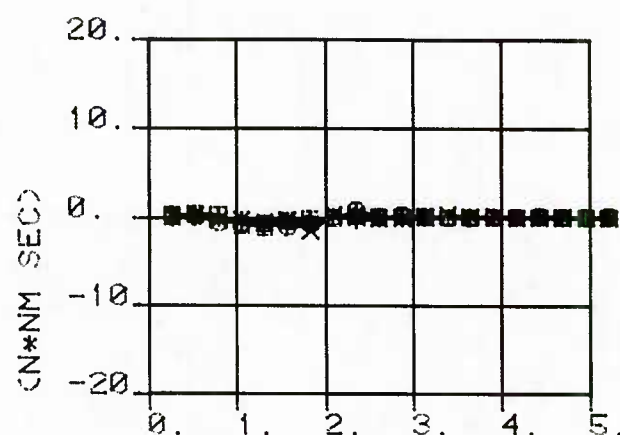
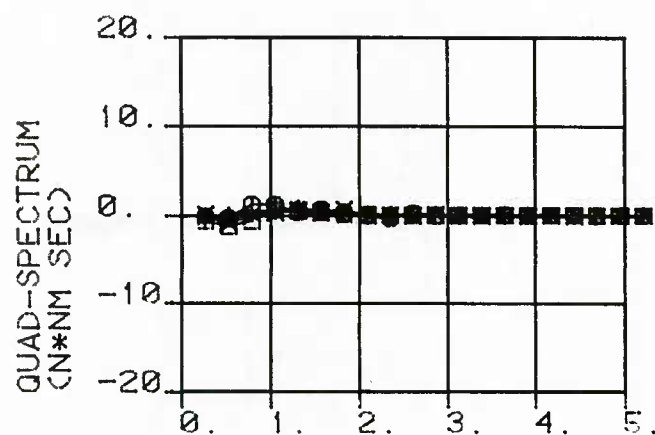
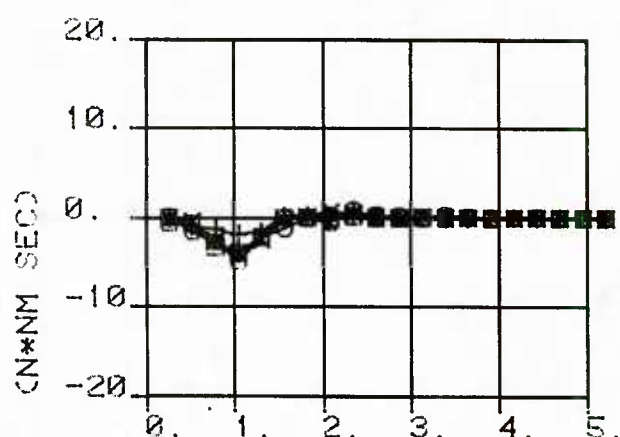
## CROSS-SPECTRUM AND COHERENCE-FUNCTION

VEL = .22 M/SEC : SHEET A  $\square$  , SHEET B  $\circ$ VEL = .30 M/SEC : SHEET C  $+$  , SHEET D  $\times$ 

FX WITH PITCH



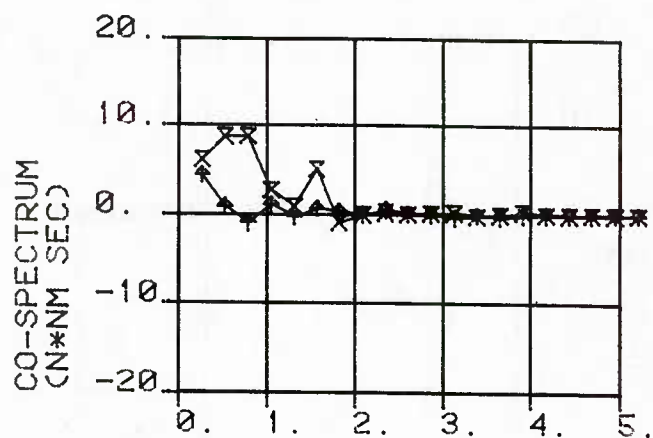
FX WITH ANG. ACC



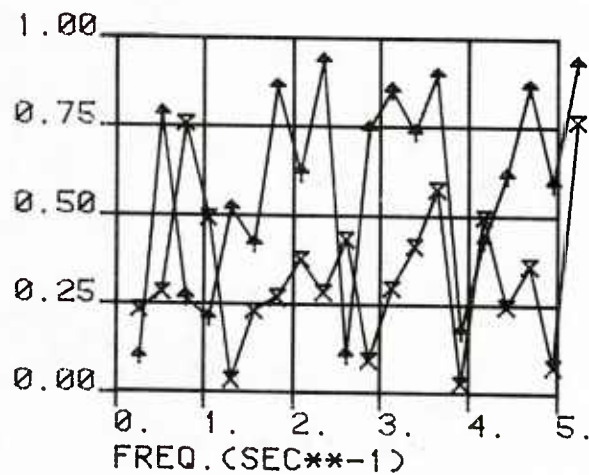
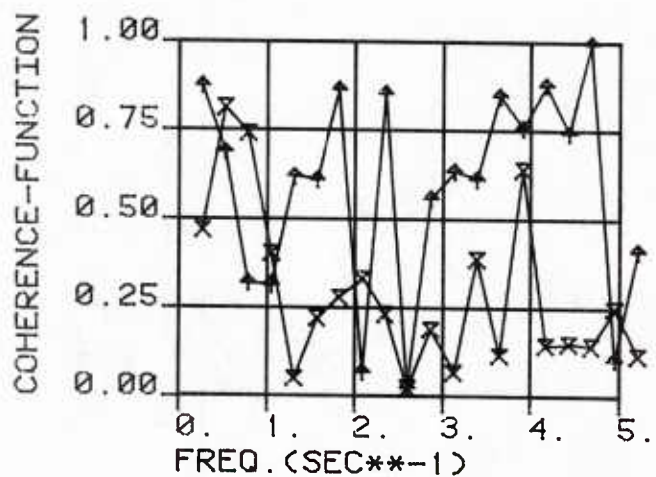
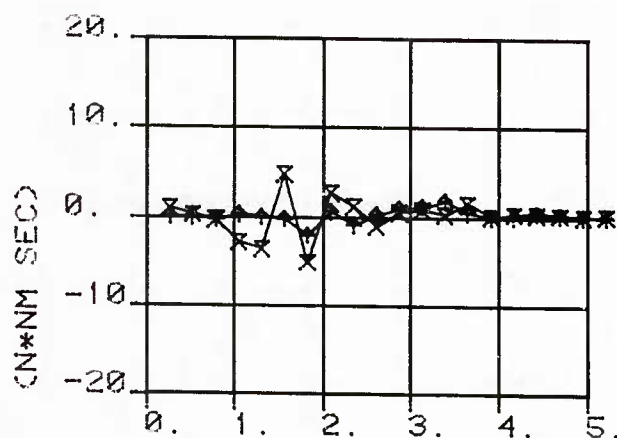
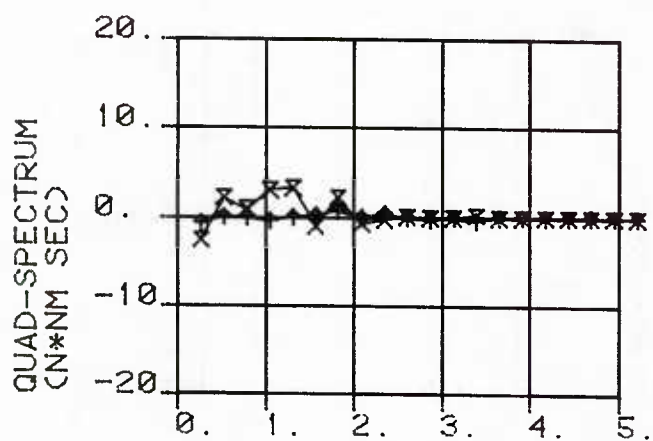
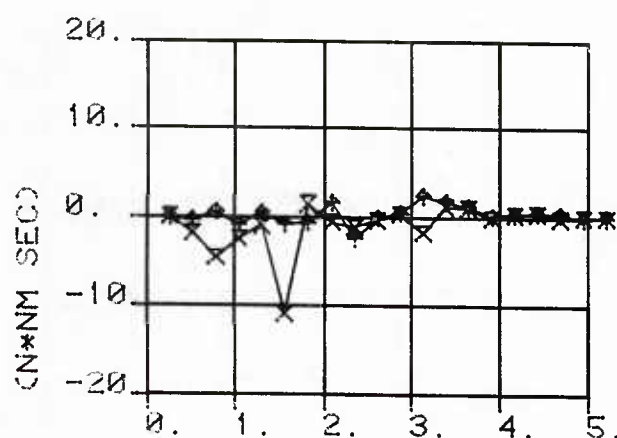
## CROSS-SPECTRUM AND COHERENCE-FUNCTION

VEL = .60 M/SEC : SHEET E  $\uparrow$ SHEET F  $\times$ 

FX WITH PITCH



FX WITH ANG. ACC

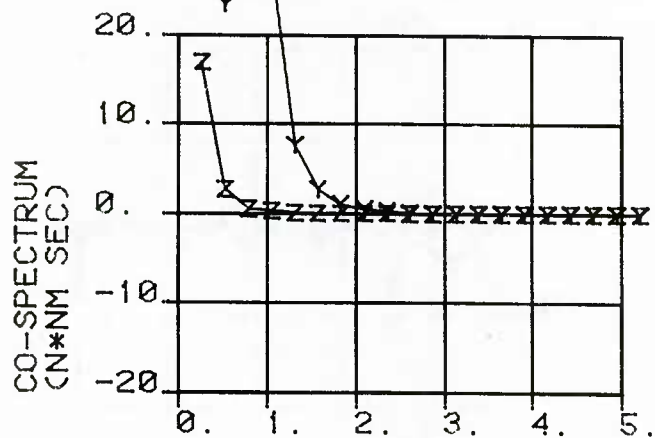


## CROSS-SPECTRUM AND COHERENCE-FUNCTION

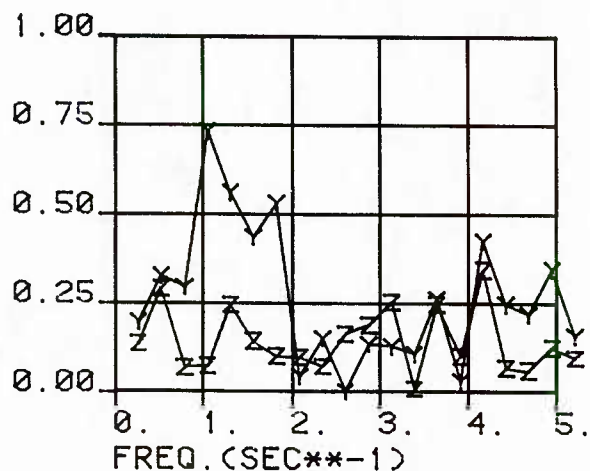
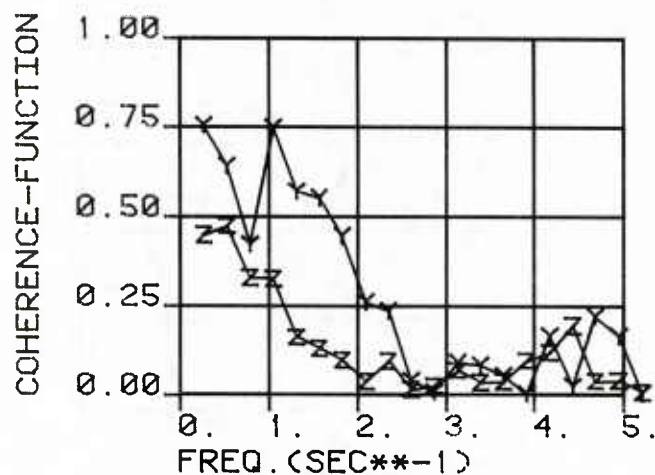
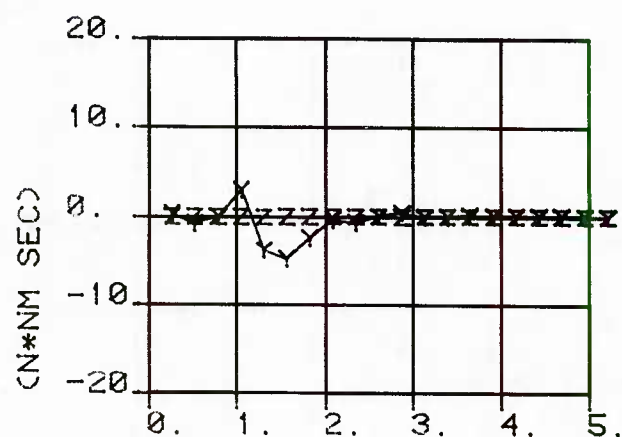
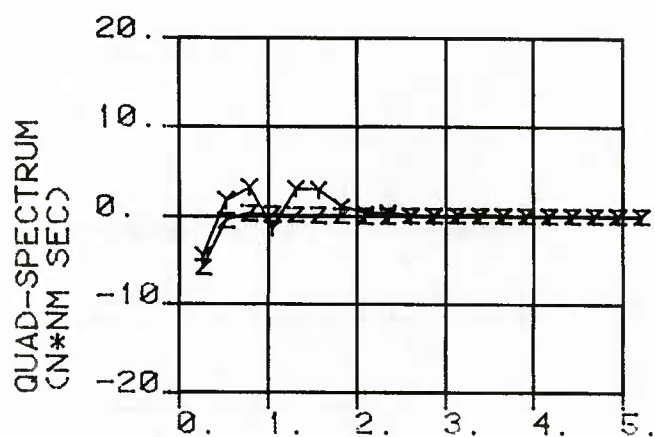
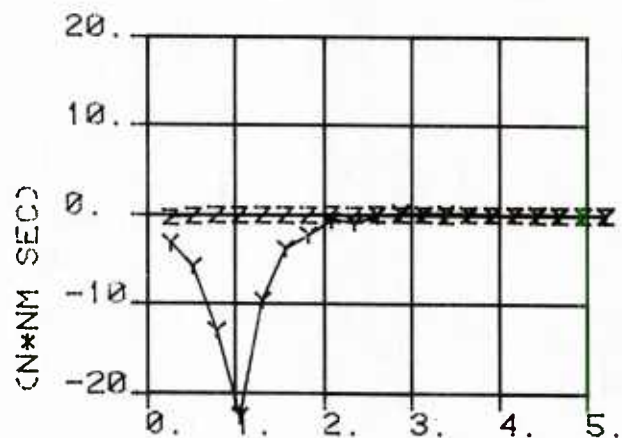
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